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ABSTRACT

This curriculum guide for the grades 7-9 mathematics program for Manitoba schools was developed as part of an overall revision of mathematics programs for Manitoba schools and is designed to articulate with the existing guide for grades K-6 and the proposed guides for grades 10-12. Included are: (1) a general overview of the program, including core objectives of both the K-6 and 7-9 programs; (2) notes on program implementation; (3) the scope and sequence for grades 6-9; (4) individual guides for each year of the 7-9 program, with objectives and specific teaching suggestions; (5) a bibliography of teacher resources; and (6) a roster of approved textbooks. (MP)

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7-9 MATHEMATICS

1979

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This guide replaces the grades 7-9 guidelines authorized and issued from 1967 to 1971 and the revised interim version issued in 1976. It has been developed as part of an overall revision of mathematics programs for Manitoba schools and is designed to articulate with the existing guide for grades K-6 and the proposed guides for grades 10-12. The final form is based on the interim document and incorporates the suggestions and input obtained as feedback from those teachers who field tested the interim guide. This curriculum guide is the framework upon which all grades 7-9 teachers are to build their mathematics program. It will become the approved guide for grades 7 and 8, effective September, 1979, and for grade 9 not later than September, 1980.

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The members of the Mathematics 7-9 Curriculum Committee are as follows:

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I. GENERAL OVERVIEW OF THE PROGRAM

This section provides a general description and summary of the uses of mathematics, the general goals, and the specific objectives of the K-9 mathematics program. Schools may find this general overview particularly useful in describing the program to parents.

A. General Information

The adolescent years, beginning around age eleven and ending at age fifteen or sixteen, is a distinctive period of human development. It is the period of transition from childhood to adulthood. The rate of physical, mental, emotional, and social development varies widely in students of this age group.

During this period, physical development proceeds rapidly and coupled with the onset of puberty, may give rise to coordination problems, lethargy, and undue concern about peer relationships. The uncertainty, occurring in students because of the changes which they are undergoing, can provide parents and the school with an opportunity to establish relevant communication with these students. Being receptive, providing supportive comments or guidance and generally being concerned about the students are important to maintain meaningful communication.

Students in grades 7-9 learn best through active personal involvement. According to developmental learning theory, students progress through three stages of learning; concrete (manipulative), semi-concrete (pictorial) and abstract (symbolic). Mathematics concepts should develop according to these stages of learning. If the concept is such that first hand experience is impractical or if previous experiences have developed the underlying concepts, then initial activities may be at the semi-concrete or abstract levels. As students progress, the time devoted to concrete and semi-concrete activities thus decreases while the time spent in more abstract activities increases.

A balance should be maintained between basic skill development and an understanding of the structure and vocabulary of the discipline of mathematics. To achieve this balance, terminology and technical definitions have been de-emphasized, understanding using concrete materials stressed, and appropriate practice recommended.

Parents should avoid transmitting to a student any negative feelings about mathematics which they may have experienced in their own schooling as this does not contribute to the fostering of a positive attitude.

Home and school, working together while being mutually supportive, can be a most effective force in producing and maintaining the best possible mathematics program..

B. Use of Mathematics

The study of mathematics is important for utilitarian purposes. It has many applications in everyday life and in specialized scientific work. A knowledge of mathematics is necessary to enable students to attain an awareness and an understanding of their environment in order to maximize their self-potential and enjoy a fuller life.

With this in mind, the Mathematics Working Party (K-12) suggests that the basic uses for mathematics are as follows:

1. Communication - Mathematical language is an integral part of a student's essential communication process. Basic concepts and terminology must be learned and understood in order to be able to communicate effectively. Mathematical concepts should be introduced to develop a language which is used to express relationships observed in the real world. At a later stage, symbols should be introduced to record these relations. Technical terminology should not be introduced unless it is genuinely required. The language used must be appropriate to the level of the students. Excessive logical rigor should be avoided;

2. Tool - After a mathematical system has been introduced as a language, it should be used as a tool to answer practical questions of genuine interest. For a student to function in society, basic skills in arithmetic, algebra, geometry and statistics are necessary. These would include fundamental operations and information processing involving collection, organization and interpretation of data. Problem solving is vitally important. It involves the application of acquired skills and concepts in familiar and unfamiliar contexts. Students should be capable of identifying problems and of employing an orgnaized process leading to a solution. Consumer awareness should be fostered through selecting realistic problems relating to the students' environment;

3. Appreciation - Some consider mathematics to be an aesthetic discipline like music. In the middle years, students can develop an appreciation for mathematics through the discovery of patterns in numbers and in man-made and naturally occurring designs. Students should be encouraged to reproduce and create their own patterns and designs. Methodologies and techniques conducive to the development of positive inquiring attitudes toward mathematics should be employed.

The development of mathematics has been most vigorous and successful when each of the three uses has received appropriate consideration. The degree of emphasis on each will vary considerably depending upon the interest and the ability of the student and the nature of the topic. These uses should be incorporated into the mathematics curriculum in order to meet the needs of the student as a better informed and more involved member of society.

C. General Goal of Mathematics

The aim of mathematics is to foster maximum and continuous development of each student's potentialities in terms of the three areas of behavioural growth: the cognitive (factual), the affective (feelings, perceptions), and the psychomotor (movements).

Growth in these behaviours, which will help the student relate to his/her physical and social environment, will result from his/her using mathematical content to develop processes such as the following:

1. The process of communicating - using mathematical language and symbolism.
2. The process of recall - a means of developing cognitive (factual) skills.
3. The process of conceptualizing mathematical principles.
4. The process of seeing relationships in mathematics.
5. The process of generalizing - finding common elements among mathematical concepts.
6. The process of making applications - applying generalizations to new situations.
7. The process of self-direction - searching for mathematical ideas.
8. The process of discovery - recognizing mathematical ideas.
9. The process of creativity - generating mathematical ideas.
10. The process of appreciating the place of mathematics and its widespread applications in our daily activities.
11. The process of problem solving - identifying, hypothesizing, interpreting, relating, evaluating, concluding.

Mathematical content study will promote growth in these processes which in turn will promote growth in the three behavioural areas.

D. CORE OBJECTIVES GRADES K-6

Core Objectives - Kindergarten

As a result of the program, the student should be able to:

1. In informal activities sort objects into sets:
   a. according to colour
   b. according to size
   c. according to shape
   d. for other reasons
2. Compare two objects and show the difference or similarity:
   a. size
   b. colour
   c. texture
   d. mass
   e. bigger than
   f. smaller than
   g. other
3. In formal activities sort objects according to:
   a. one specific attribute
   b. two attributes
4. Compare two sets of objects to determine which has more, less, as many as.
5. Reproduce a set that has as many objects as, more objects than, less or fewer objects than the given set.
6. Find equivalent sets or select from many sets those sets that have as many things as a given set by matching.
7. Reproduce a given pattern concretely.
8. Continue a given pattern concretely.
9. Create a pattern concretely or pictorially.
10. Arrange a set of objects in order by size or by length.
11. Associate specific events with appropriate parts of the day.
12. Sequence events as they occur during the day.
13. Begin to get ready for the study of divisions of the year.
14. Begin to get ready for the concepts of yesterday and today.

Core Objectives - Grade 1

As a result of the program, the student should be able to:

1. Compare two objects or two sets of objects re colour, size, etc. Using objects and then pictures develop knowledge of: same, different, larger, bigger, smaller, longer, taller, shorter, wider, narrower.
2. Use and understand vocabulary of position. Using objects and then pictures develop knowledge of such terms as: above, below, over, under, before, after, first, last, between, behind, in front of.
3. Compare more than two objects or two or more sets of objects using such terms as: as many as, more than, fewer than, less than, etc.
4. Count at least up to 10 and down, and count by twos, threes, fours, and fives.
5. Determine the cardinal number of a set with 0 through 10 members (rational counting).
6. Combine two sets to form a larger set (addition) and remove a subset from a larger set (subtraction).
7. Order sets in increasing or decreasing order.
8. Reproduce sets equivalent to (as many as) given sets or named sets.
9. Relate a set to its numeral (symbol) and number.
10. Order numbers in increasing and decreasing order.
11. Understand the property of zero (empty set), e.g., $5 + 0 = 5$.
12. Solve addition number sentences (join two sets) and recognize that addition is commutative, e.g., $5 + 4 = 4 + 5$.
13. Solve subtraction number sentences (remove subsets) or find the difference between two sets.
14. Decide the appropriate operation needed to solve a simple problem.
15. Write numerals (symbols) for numbers 1-9 in sequence after developing and understanding the number concept.
16. Write number sentences to represent number stories.
17. Use numbers 0-10 and develop the number stories for each and learn to use vertical notation for addition and subtraction.
18. Learn to use and understand the symbols +, -, and =.
19. Understand the relation between addition and subtraction as inverse operations.
20. Begin to recall the basic facts for addition and subtraction.
21. Reproduce, continue, and create concrete, pictorial, or action patterns.

22. After having many concrete number experiences after talking about them, proceed slowly with written work using abstract mathematical symbols.
23. Begin to develop the concept of multiplication and division through concrete experiences.
24. Read and write numerals up to 99.
25. Develop an understanding of fraction ideas through activities with concrete materials, e.g., $\frac{1}{2}$, $\frac{1}{4}$.
26. Become aware of and be able to compare and classify 3-D solid geometric shapes, e.g., ball, can, box, etc.
27. Become aware of and be able to compare and classify 2-D shapes, e.g., square or rectangle from a box shape, circle from the top of a can, triangle from certain prisms or pyramids.
28. Understand graphs as a means of visually depicting information using concrete graphs, pictographs, 3-D bar graphs.
29. Recognize coins, e.g., 1¢, 5¢, 10¢, 25¢.
30. Name the days of the week, months of the year.
31. Tell time to the hour.
32. Check off the date on a calendar.
33. Use arbitrary units such as hands, blocks, etc., and standard units such as centimetre and metre to measure and compare length.
34. Develop an awareness of area, volume, capacity, and mass through activities.
35. Solve simple problems through experiences with concrete situations.

Core Objectives - Grade 2

As a result of the program, the student should be able to:

1. Count up to 100 and skip-count by twos, fives, tens and later by fours and threes. Use concrete experiences.
2. Understand place value (11-19) and read and write numerals.
3. Study whole numbers 0-20 individually and relate each to previously studied numbers.
4. Rename numbers as addition sentences, e.g., $12 = 10 + 2$.
5. Understand and apply the associative property, e.g., $(9 + 1) + 2 = 9 + (1 + 2)$ - use concrete material.
6. Understand and apply the commutative property, e.g., $3 \times 2 = 2 \times 3$ - use concrete material.
7. Understand the property of zero in addition, e.g., $3 + 0 = 3$.
8. Read, write, and interpret horizontal and vertical notation.
9. Understand subtraction as the removal of a subset and as the inverse of addition.
10. Compare sets (one-to-one correspondence) and state the relationship in as many ways (number sentences) as possible.
11. Understand the property of zero in subtraction, e.g., $19 - 0 = 19$.
12. Improve proficiency (speed and accuracy) with basic addition and subtraction facts.
13. Develop multiplication facts up to 20 by joining equivalent sets, by using arrays, by repeated addition on a number line, and by skip-counting.
14. Understand division as finding the number of equivalent subsets in a given set, as the inverse of multiplication, and as repeated subtraction on a number line.
15. Learn to use and understand the symbols "x" and "÷".
16. Increase proficiency with multiplication and division facts up to 20.
17. Find and identify $\frac{1}{2}$ and $\frac{1}{4}$ of a concrete thing, and $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ of a set, and $\frac{1}{10}$ as it relates to measurement.
18. Identify, reproduce, continue, and/or create concrete, pictorial or action patterns.
19. Understand place value up to two places.
20. Read and write the numerals up to 100.
21. Apply counting by tens to real situations, e.g., 96 pennies as 9 tens and 6 ones.
22. Relate a number to other numbers before and after or $<$ and $>$ (ordering).
23. Express a number in expanded form orally and in writing as having tens and ones.
24. Round off or estimate to the nearest ten.
25. Add a two-digit and a one-digit number without regrouping (column addition).
26. Recognize and classify 3-D shapes.
27. Analyze and classify 3-D shapes by tracing faces, and by counting faces, corners, edges.
28. Estimate in measurement.
29. Measure lengths with arbitrary units and with standard units, e.g., metres and centimetres.
30. Understand and know the relationship between metres and centimetres.

31. Be aware of and begin to develop an understanding of area and volume using non-standard units in covering up and filling up activities.
32. Become aware of capacity through measurement activities using litre and half-litre.
33. Use the kilogram as a unit of measuring mass, e.g., weighing themselves and record or graph the results.
34. Understand such units as morning, noon, afternoon, evening, night, and be able to tell time by the hour, half-hour and quarter-hour.
35. Read the Celsius thermometer and discuss weather in relation to comfort temperatures.
36. Recognize Canadian coins and the one dollar bill through involvement in activities and problems involving addition and subtraction up to 99¢.
37. Develop an understanding of concrete graphs, concrete bar graphs, and pictographs through ongoing graphing activities in several subject areas.
38. Develop problem-solving skills through daily experiences in all subject areas. Begin with oral problems and illustrate answers with concrete materials, e.g., on flannelboard, on magnetic board, etc., and progress to expressing the mathematical sentence orally and in writing.
39. Solve written problems after having acquired sufficient reading and oral problem-solving skills.

Core Objectives - Grade 3

As a result of the program, the student should be able to:

1. Read and write, and know the place value of each digit in numerals 0-999.
2. Know the meaning of and be able to use in number sentences such symbols as: +, -, x, ÷, =, $>$, $<$.
3. Round off numbers to the nearest hundred.
4. Read and write Roman numerals to XX.
5. Write a number in expanded form.
6. Increase proficiency with the addition and subtraction facts up to 20 and the multiplication and division facts up to 40.
7. Rename a number in many ways.
8. Add and subtract multiples of 10 and 100, and columns of 3-digit numbers with and without regrouping.
9. Do multiplication and division questions involving 10 and 100 and multiply a 1-digit by a 2-digit number and divide a 2-digit by a 1-digit number.
10. Identify, continue, reproduce, and create patterns using concrete and pictorial materials.
11. Identify fractions as part of a unit or part of a set: $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{1}{10}$ related to metric measure and $\frac{1}{100}$ related to money.
12. Describe 3-D and 2-D shapes in the environment re faces, edges, corners, etc., and sort and classify similar shapes.
13. Locate (not define) and/or illustrate angles, parallel lines, a curved line, or a straight line.
14. Recognize many-sided plane shapes, e.g., stop sign, in the environment.
15. Reproduce a square, a rectangle, and a line segment.
16. Locate lines of symmetry.
17. Estimate and then measure a distance in non-standard and in standard metric units and know the relationships:
    1 m = 100 cm
    1 m = 10 dm
18. Estimate and then find the area of a given 2-D space by covering it up with non-standard or standard 2-D units of area, e.g., cm^2 or m^2.
19. Estimate and then find the volume of a given 3-D space by filling it up with non-standard or standard 3-D units of volume, cm^3.
20. Estimate and measure the capacity of a container in appropriate units: litre, half-litre, millilitre.
21. Estimate and measure the mass of various things using appropriate units, e.g., kilogram, gram.
22. Read the time down to 5-minute intervals and know the divisions of the day, the days of the week, the months of the year, and concepts of yesterday, today, and tomorrow.
23. Read a Celsius thermometer.
24. Solve problems involving addition and subtraction related to money.
25. Construct and interpret graphs using real things, using objects to represent real things, using pictographs, using bar graphs, all involving a one-to-one representation.
26. Interpret and solve a word problem orally and in writing.
27. Apply the concept of estimation (including rounding off) in all problem solving and measurement situations.

Core Objectives - Grade 4

As a result of the program, the student should be able to:

1. Read and write, and know the place value of the digits in

up to six-digit numbers.
2. Write numbers in expanded notation.
3. Rename a number in many ways.
4. Round off whole numbers up to six digits.
5. Know the meaning of and be able to use in number sentences such symbols as: +, -, x, ÷, =, < , > , ≠.
6. Recognize I, V, X, L, C, D, M and read and write numbers up to C.
7. Identify, construct, and continue simple concrete, pictorial, or numerical patterns, e.g., odd numbers, even numbers, factors, primes, composites.
8. Recall with proficiency all basic facts for addition, subtraction, multiplication and division.
9. Relate commutative property, associative property, and property of zero to addition and subtraction as applicable.
10. Add and subtract with and without regrouping with up to three 4-digit addends for addition and subtraction with 4-digit numbers.
11. Round off to tens, hundreds and thousands for estimation.
12. Check addition and subtraction.
13. Apply the commutative property, associative property, distributive property, property of zero, and property of one to multiplication.
14. Multiply whole numbers involving up to a 4-digit factor by a 2-digit factor.
15. Divide up to a 3-digit dividend by a 1-digit divisor, with and without remainder. Lead to the standard algorithm.
16. Determine sets of equivalent fractions.
17. Add fractions having like denominators with sums less than one and sums equal to one.
18. Subtract proper fractions with like denominators and subtract a proper fraction from one.
19. Express 10's and 100's as decimals.
20. Tell time to the nearest minute and use terms a.m. and p.m.
21. Add and subtract units of time with and without regrouping.
22. Read the Celsius thermometer above and below zero.
23. Know relationships among m, dm, cm, km, and measure lengths of objects using appropriate units. Determine perimeter by measuring.
24. Find the area of a rectangle (no formula).
25. Determine volume by filling up a container with cubic units (no formula).
26. Determine the capacity of a container using appropriate units (L, mL) and know the relationship between litre and millilitre.
27. Add and subtract capacity measures with and without regrouping.
28. Determine the weight of an object using the appropriate units (kg, g).
29. Recognize and identify 3-D and 2-D shapes in the environment (natural and man-made).
30. Test plane shapes for congruence.
31. Locate axes of symmetry in regular plane shapes.
32. Identify angles as right angles or non-right angles.
33. Illustrate the position of an object (shape) after a slide, flip, or turn.
34. Construct and interpret, involving a one-to-one or a one-to-many representation:
    a. concrete graphs
    b. pictographs
    c. bar graph (horizontal or vertical)
    d. line graph
35. Solve oral and written one-step word problems. Begin developing a procedure for solving all problems.

Core Objectives - Grade 5

As a result of the program, the student should be able to:

1. Read and write base ten whole numbers up to nine digits, including zero as a place holder, and decimals up to three places, and identify the value of any digit in the number.
2. Write whole numbers having up to nine digits and decimals up to three places in expanded form.
3. Round off whole numbers to any of the digits in up to a 9-digit number and decimals to whole, tenths, hundredths, in order to estimate sums, differences, products, and quotients.
4. Use appropriate symbols (<, =, ≠, >) to make number sentences true for either equalities or inequalities.
5. Read and write Roman numerals from I to M and translate to Hindu-Arabic numerals and vice versa.
6. Identify, construct, and continue simple concrete, pictorial, or numerical patterns.
7. Identify and illustrate even and odd numbers, factors and multiples, primes and composites, common factors, common multiples, lowest common multiple, and rules of

divisibility for 2, 5, and 10.

8. Recall with accuracy and proficiency all basic number facts for addition, subtraction, multiplication, and division.
9. Add and subtract ten and multiples of ten to or from a given number, and multiply and divide a number by ten or multiples of ten orally and in writing.
10. Relate the commutative property, the associative property, the property of zero, and the property of one to the operations of addition, subtraction, multiplication, and division of whole numbers, fractions, and decimals as applicable.
11. Add and subtract with and without regrouping using expanded notation, place value grid, and column and horizontal formats for up to four 5-digit numbers or five 4-digit numbers and apply to adding and subtracting money.
12. Begin to use the standard multiplication algorithm to multiply up to four digits by three digits and apply to problems involving money.
13. Begin to use the standard algorithm to divide up to 6-digit dividend by a 2-digit divisor with and without remainders and apply to money problems.
14. Check the addition, subtraction, multiplication, and division operations.
15. Find the average of a group of three numbers.
16. Illustrate concretely or pictorially the meaning of a fraction, and know the terms numerator, denominator, proper fraction, improper fraction, mixed number, equivalent fraction, and determine a set of equivalent fractions for a named fraction.
17. Reduce a fraction to lowest terms, raise a fraction to higher terms and compare fractions by changing to the same denominator and ordering from smallest to largest or vice versa.
18. Rename an improper fraction as a mixed number or a whole number and vice versa.
19. Add proper fractions with like or unlike denominators, and mixed numbers and proper fractions using either vertical or horizontal format.
20. Subtract: proper fractions, proper fractions from mixed numbers with like denominators, a proper fraction or a mixed number from a whole number, a mixed number from a mixed number with like or unlike denominators, but with no regrouping.
21. Express decimals (up to three places) in fractional form in lowest terms and vice versa.
22. Add and subtract decimals involving tenths, hundredths, and thousandths, using either vertical or horizontal notation.
23. Know the pertinent relationships among second, minute, hour, day, week, month, year, decade, and century, record the date in SI format, read the time on a 12-hour clock, a 24-hour clock, a digital clock, and add and subtract units of time.
24. Read the Celsius thermometer above and below zero.
25. Know the relationships among kilometre, metre, decimetre, centimetre, and millimetre, use the appropriate units to measure linear distances, add and subtract linear measures with and without regrouping, and explore and measure perimeters (no formula).
26. Know the relationships among the common units of area and use the appropriate units to determine or measure the area of various geometric shapes (no formula).
27. Know the relationships among the common units of volume and use the appropriate units to determine or measure the volume of various cubic structures (no formula).
28. Know the relationships among the common units of capacity and use the appropriate units to determine or measure the capacity of various containers (no formula).
29. Know the relationships among the common units of mass or weight and determine or measure the mass of various objects.
30. Describe, illustrate, construct, and label a line segment, and identify open and closed curves.
31. Identify, locate, and construct (not compass and ruler constructions) parallel and perpendicular lines.
32. Test given line segments and shapes for congruency and reproduce line segments and shapes congruent to given line segments and shapes.
33. Compare given angles with right angles and measure angles using an arbitrary angle as the unit of measure.
34. Classify three-, four-, five-, six-, and eight-sided regular polygons according to the number of equal sides and angles.
35. Classify triangles as right angled or non-right angled and construct right triangles using graph paper, geopaper, geoboard, and paper folding.
36. Test shapes for and identify lines of symmetry in nature and in the environment.
37. Identify a motion as a slide, flip, or turn. Construct images of shapes after a slide, flip, or turn, and find examples of slides, flips, or turns in the environment.
38. Construct, interpret, title, and report on a concrete graph, a pictograph, a bar graph, a line graph, and locate points represented by ordered pairs or number

pairs in the first quadrant.

39. Solve oral, computational and word problems involving one- and two-steps. Continue to refine the procedure for solving all problems. This may apply to whole numbers, fractions, and decimals.

Core Objectives - Grade 6

As a result of the program, the student should be able to:

1. Read and write base 10 whole numbers up to ten digits and decimals up to three places.
2. Write numbers in expanded form.
3. Interpret and write numbers (tens) in exponential form when writing numbers in expanded form.
4. Rename numbers in many ways.
5. Round off whole numbers up to ten digits, and decimals up to any one of three places.
6. Write appropriate symbols to make number sentences of equalities and inequalities true.
7. Identify, construct, complete, and create concrete patterns, pictorial patterns, and numerical patterns.
8. Identify and illustrate even and odd numbers, factors multiples, common factors, greatest common factor (G.C.F.), common multiples, lowest common multiple (L.C.M.), prime numbers and composite numbers from 1-100, and rules of divisibility for 2, 4, 3, 6, 9, 5, and 10.
9. Recall with proficiency (automatic response) all basic number facts for addition, subtraction, multiplication and division.
10. Add and subtract groups of numbers with up to six digits with and without grouping, and in either vertical or horizontal format.
11. Add and subtract money involving up to six digits.
12. Round off to multiples of ten to estimate answers when adding, subtracting, multiplying and dividing.
13. Check addition, subtraction, multiplication and division.
14. Multiply and divide whole numbers using the standard algorithms with up to a combined total of eight digits in multiplication and up to a 6-digit dividend by a 3-digit divisor with and without remainders.
15. Perform multiplication and division operations involving money by converting dollars to cents to avoid decimals.
16. Find the arithmetic mean or average of at least three numbers.
17. Find sets of equivalent fractions, reduce a fraction to lowest terms, raise a fraction to higher terms, compare fractions having like or unlike denominators, order fractions in increasing or decreasing order.
18. Add (up to four addends) and subtract proper fractions or mixed numbers with like or unlike denominators with and without regrouping using the vertical or horizontal format.
19. Multiply a fraction by a whole number and a mixed number by a whole number and vice versa.
20. Understand and read and write decimals up to three places and rename as fractions in reduced form and vice versa.
21. Express a decimal and a fraction as a percent and rename a percent as a fraction and a decimal.
22. Round off decimals to wholes, tenths, or hundredths.
23. Add and subtract decimals, using the vertical or horizontal format, and involving combinations of tenths, hundredths and thousandths.
24. Multiply a decimal by a whole number and divide with up to a 3-digit decimal divisor by converting to whole numbers.
25. Apply addition, subtraction, multiplication, and division of decimals to one- and two-step problems.
26. Recognize, interpret, and use different ratio formats and recognize a proportion as a comparison of two ratios.
27. Apply the commutative property, associative property, distributive property, the property of zero, and the property of one where applicable to the operations of addition, subtraction, multiplication and division re whole numbers, fractions, and decimals.
28. Maintain the pertinent relationships among second, minute, hour, day, week, month, year, decade and century, record the date in SI format and read the time on a 12-hour, 24-hour, and digital clock.
29. Solve money problems involving addition, subtraction, multiplication, and division.
30. Read the Celsius thermometer above and below zero.
31. Know the relationships among kilometre, metre, decimetre, centimetre and millimetre, and use appropriate units to measure linear distances.
32. Using appropriate units, determine the area of various shapes and discover the formula for rectangular shapes ($A = L \times W$).
33. Using appropriate units, determine the volume of various objects and explore the relationship between volume and capacity.
34. Using appropriate units, determine the capacity of a container.
35. Using appropriate units, measure the mass of an object.

36. Be aware of the basic concepts pertaining to 3-D (solid) and 2-D (plane) shapes and identify open and closed curves.
37. Name the parts of an angle, label an angle, measure an angle with a protractor, and classify angles according to their size.
38. Classify triangles, quadrilaterals, and polygons according to the number of equal sides.
39. Determine the number of lines of symmetry in regular polygons.
40. Identify and name the parts of a circle.
41. Test figures for congruence.
42. Identify motions as slides, flips, or turns and indicate the position of a shape following a slide, flip, or turn, and test shapes for congruence after a slide, flip, or turn.
43. Construct, interpret, title, and report on concrete graphs, pictographs, bar graphs, and line graphs, plot number pairs or ordered pairs in the first quadrant, and interpret simple circle graphs.
44. Solve oral, computational and word problems involving one- and two-steps. The procedure should involve the process of estimating the answer, the written analysis (writing the number sentence), using the given information, the solving of the number sentence, and the comparison of the estimated and the computed answer, and the checking of the computed answer.

E. CORE OBJECTIVES GRADES 7-9

Core Objectives - Grade 7

As a result of the program, the student should be able:

1. To discuss the relative efficiency of the Hindu-Arabic number system with at least one other number system.
2. To identify the set of natural or counting numbers (N).
3. To apply the Property of One for Multiplication.
4. To illustrate the set of whole numbers (W) as an extension of the set of natural numbers (N).
5. To apply the Property of Zero for Addition and Multiplication.
6. To illustrate the Closure Property as it applies to whole numbers.
7. To explain and to illustrate the structure of the Hindu-Arabic or decimal numeration system.
8. To rewrite numbers in expanded format, without using exponents, and vice versa.
9. To round off whole numbers.
10. To estimate, to calculate and to check the solution to any problem involving the operations of addition, subtraction, multiplication and division of whole numbers.
11. To estimate and to calculate the solution to number phrases involving several operations with or without brackets.
12. To demonstrate an understanding of the meaning of "power", "base", and "exponent".
13. To convert a power to a simple number and vice versa.
14. To apply the concept of exponents to expanded notation.
15. To perform simple calculations involving powers using the order of operations.
16. To solve and to check word problems pertaining to any of the preceding concepts or skills which involve whole numbers.
17. To express word phrases as number phrases or expressions and vice versa.
18. To evaluate open number phrases by substitution.
19. To solve simple one variable equations.
20. To translate simple word problems to equations and then to solve the equations.

21. To list the set of factors of a given number.
22. To list the set of multiples of a given number.
23. To determine the greatest common factor (GCF) of a set of composite numbers.
24. To determine the least common multiple (LCM) of a set of numbers.
25. To explain and to give examples of prime and composite numbers.
26. To write a composite number as a product of prime numbers.
27. To identify and to give examples of numbers that are relatively prime.
28. To demonstrate an understanding of a fraction as:

    a. a part of a whole,
    b. a part of a set,
    c. a comparison (ratio),
    d. a division.

29. To identify the numerator and denominator of a fraction.
30. To locate fractions on a number line and to identify the fractional equivalent of a point on a number line.
31. To identify proper and improper fractions.
32. To identify mixed fractions or mixed numbers.
33. To identify equivalent fractions.
34. To generate sets of equivalent fractions.
35. To change improper fractions to mixed fractions and vice versa.
36. To reduce fractions to lowest terms.
37. To compare fractions.
38. To add and to subtract:

    a. proper fractions,
    b. mixed fractions.

39. To multiply:

    a. proper fraction times whole number,
    b. mixed fraction times whole number,
    c. proper fraction times proper fraction,
    d. proper fraction times mixed fraction,
    e. mixed fraction times mixed fraction.

40. To find the multiplicative inverse or reciprocal of any fraction or whole number.

41. To divide:

    a. proper fractions,
    b. mixed fractions.

42. To solve and to check word problems involving fractions.
43. To read decimal fractions to at least three decimal places.
44. To locate decimal fractions on a number line.
45. To write a decimal fraction in expanded notation without exponents.
46. To write a decimal fraction in expanded notation using only positive exponents.
47. To round off decimal fractions to the nearest whole number.
48. To round off decimal fractions to the nearest tenth, hundredth or thousandth.
49. To change a fraction to a decimal fraction.
50. To rewrite a terminating decimal fraction as its fractional equivalent.
51. To add and to subtract decimal fractions with any number of decimal places.
52. To multiply or to divide a decimal fraction by a power of ten (i.e., 10, 100, 1000, etc.).
53. To multiply a decimal fraction by a whole number other than a power of 10.
54. To multiply a decimal fraction by a decimal fraction with any number of decimal places.
55. To divide a decimal fraction by a power of ten.
56. To divide a decimal fraction by a whole number other than a power of ten.
57. To divide a decimal fraction by a decimal fraction involving any number of decimal places.
58. To solve and to check word problems involving decimal fractions.
59. To demonstrate an understanding of ratio as a comparison of two numbers, using the notation a to b or a:b or $\frac{a}{b}$.
60. To set up a ratio between two quantities having the same unit of measure.
61. To set up a rate between two quantities having different units of measure.
62. To demonstrate an understanding of a proportion as equivalent ratios.
63. To apply and to solve problems relating to:
    a. scale drawings,
    b. map reading,
    c. unit pricing,
    d. miscellaneous problems.
64. To demonstrate an understanding of percent as a special ratio of a number compared to 100.
65. To change fractions to percent and vice versa.
66. To change decimal fractions to percent and vice versa.
67. To calculate a specific percentage of a given number.
68. To calculate what percentage one number is of another number.
69. To calculate provincial sales tax on a specified purchase.
70. To identify examples of line segments from the environment.
71. To estimate the length of a line segment in millimetres and in centimetres.
72. To use a metric tape or ruler to measure line segments to the nearest millimetre, centimetre or metre.
73. To classify lines as being horizontal, vertical or oblique.
74. To identify a pair of lines as being parallel or perpendicular.
75. To identify the lines of symmetry for segments.
76. To determine the perimeter of any polygon through measurement or through computation when given the lengths of the sides.
77. To determine the area of any square, rectangle or parallelogram.
78. To identify the vertex and sides of an angle.
79. To name an angle using a proper notation.
80. To measure the size of an angle, whose measure is less than 180^{0}, using a protractor.
81. To draw any angle, whose measure is less than 180^{0}, using a protractor.
82. To identify angles according to their size, as being acute, right, obtuse or straight.
83. To identify the line of symmetry for any angle.
84. To use the proper notation to name a triangle.
85. To determine the perimeter of a triangle.
86. To measure each of the angles of a triangle using a protractor.
87. To classify triangles by sides and angles.
88. To identify the lines of symmetry for triangles.
89. To identify rotational or point symmetry.
90. To determine the area of a triangle.
91. To identify and to illustrate slides (translations),

flips (reflections), turns (rotations) and combinations of these rigid motions.

92. To interpret and to construct a pictograph.
93. To interpret and to construct a bar graph.
94. To interpret and to construct a line graph.
95. To locate points and to identify coordinate pairs in the first quadrant.

Core Objectives - Grade 8

As a result of the program, the student should be able:

1. To demonstrate proficiency in performing operations with natural and whole numbers:
   a. by determining the next number in a given number sequence;
   b. by calculating the arithmetic mean and median for a set of data.
2. To test any number for divisibility by 2, 3, 4, ..., 10.
3. To demonstrate proficiency in the fundamentals:
   a. place value,
   b. expanded notation,
   c. rounding-off and estimation,
   d. prime and composite numbers,
   e. LCM and GCF for two or more whole numbers.
4. To identify the set of integers.
5. To identify the additive inverse (opposite) of any integer.
6. To add and to subtract integers.
7. To multiply and to divide integers.
8. To solve simple problems involving:
   a. income and expenditures,
   b. bank balances,
   c. temperatures,
   d. weights.
9. To change word phrases to open number phrases and vice versa.
10. To evaluate open number phrases by substitution in:
    a. one variable expressions,
    b. two variable expressions.
11. To add and to subtract like terms in one variable.
12. To simplify expressions involving both adding and subtracting.
13. To solve equations in one variable where the variable appears on only one side of the equation.
14. To translate a word problem into an equation and then to solve the equation.
15. To identify rational numbers.
16. To change a fraction to a terminating or a repeating decimal fraction by division or by using equivalent fractions.
17. To change a terminating decimal fraction to a fraction.
18. To change a repeating decimal fraction to a fraction using the "pattern of nines".
19. To perform the four basic operations using rational numbers.
20. To set up a ratio or a rate between two quantities.
21. To use proportions as equivalent ratios in solving problems.
22. To define percent as a special ratio.
23. To convert:
    a. fractions to percents and vice versa,
    b. terminating decimal fractions to percents and vice versa,
    c. repeating decimal fractions to percents and vice versa.
24. To round-off percents to the nearest whole percent.
25. To estimate a percentage of a number.
26. To calculate a specific percentage of a given number.
27. To calculate what percentage one number is of another number.
28. To calculate a number when a percentage of it is known.
29. To apply the concepts of ratio and percentage relating to:
    a. provincial sales tax,
    b. commission,
    c. simple interest,
    d. discount,

e. markup and markdown,
f. scale drawings,
g. recipes,
h. mixtures.

30. To perform linear and angular measurement.
31. To determine the perimeter of any polygon.
32. To determine the area of any square, rectangle and parallelogram.
33. To demonstrate an understanding of the concept of volume and to calculate the volume of rectangular solids (i.e., cubes and cuboids).
34. To solve problems related to perimeter, area and volume.
35. To identify squares, rectangles, parallelograms and kites.
36. To determine the lines of symmetry for various quadrilaterals.
37. To determine the rotational symmetry for various quadrilaterals.
38. To identify the centre, radius, diameter, arc, sector, and circumference of a circle.
39. To name related three-dimensional objects such as spheres, cylinders and cones.
40. To draw a circle using compasses given a specific length for the radius or the diameter.
41. To determine the ratio of the circumference of a circle to its diameter and to establish the circumference formula.
42. To determine the area of a circle.
43. To determine the surface area of right circular cylinders:

    a. without ends,
    b. with ends.

44. To interpret circle graphs.
45. To construct circle graphs.
46. To identify and to illustrate slides (translations), flips (reflections), turns (rotations) and combinations of these rigid motions.
47. To identify parallel lines as slide images.
48. To identify perpendicular lines as a quarter turn about the point of intersection.
49. To demonstrate an understanding of the terminology associated with the coordinate plane.
50. To locate and plot points in all four quadrants and on the axes.

Core Objectives - Grade 9

As a result of the program, the student should be able:

1. To demonstrate proficiency in operations with whole numbers, fractions and decimal fractions.
2. To explain and to illustrate the structure of the set of integers.
3. To explain and to illustrate concepts of:

    a. additive inverse (opposite),
    b. absolute value.

4. To perform with proficiency the basic operations on integers.
5. To explain and to illustrate the meaning of rational numbers.
6. To change fractions to decimal fractions and vice versa.

    a. fractions to terminating or repeating decimal fractions;
    b. terminating decimal fractions to fractions;
    c. repeating decimal fractions to fractions.

7. To perform the four basic operations with rational numbers.
8. To solve problems relating to number operations.
9. To recognize irrational numbers.
10. To examine addition and multiplication of irrational numbers for closure.
11. To demonstrate an understanding of the definition of the set of real numbers as the union of rational and irrational numbers.
12. To demonstrate an understanding of the relationship between squares and square roots.
13. To find the exact square root of a perfect square.
14. To find the approximate square root of a natural number using:

    a. square root tables,
    b. a calculator.

15. To demonstrate an understanding of the concept of the Pythagorean relation.
16. To solve word problems involving the Pythagorean relation.
17. To use the Pythagorean relation to locate some irrational numbers on the number line.

18. To explain and to illustrate the algebraic concepts:

    a. variable,
    b. term,
    c. expression,
    d. numerical coefficient or coefficient.

19. To identify monomials, binomials and trinomials.
20. To demonstrate an understanding of and to illustrate the meaning of the terms "power", "base" and "exponent".
21. To change a numerical power with natural number exponents where the base is either integral or rational to a simple real number.
22. To change a whole number into exponential form.
23. To demonstrate an understanding of the order of operations using exponents.
24. To evaluate open expressions in one and two variables using substitution.
25. To simplify open expressions involving addition and subtraction.
26. To multiply a monomial by a monomial.
27. To multiply a monomial by a polynomial.
28. To simplify open expressions involving addition, subtraction and monomial multiplication.
29. To demonstrate proficiency in solving algebraic equations in one variable.
30. To translate word phrases into equivalent open expressions.
31. To solve algebraic word problems.

32. To perform the following basic constructions using only compasses and straightedge:

    a. to construct the perpendicular bisector of a line segment,
    b. to bisect an angle,
    c. to copy an angle,
    d. to construct a perpendicular to a line through a point on the line,
    e. to construct a perpendicular to a line through a point not on the line.

33. To construct, without the aid of a protractor, angles of:

    a. 60^0 (hence 30^0, 15^0, 120^0),

    b. 90^0 (hence 45^0, $22\frac{1}{2}^0$, 135^0),

    c. a composition of the above angles.

34. To copy triangles.

35. To construct triangles given the measures of:

    a. three sides,
    b. two sides and the included angle,
    c. two angles and any side,
    d. the hypotenuse and any side.

36. To identify and to demonstrate an understanding of the concept of opposite angles.
37. To demonstrate an understanding of the relationship between complementary angles and between supplementary angles.
38. To demonstrate an understanding of the concept of parallel lines.

39. To demonstrate an understanding of the relationship amongst angles created by parallel lines and a transversal:

    a. corresponding angles,
    b. interior or exterior alternate angles.

Option Objectives

40. To illustrate the following sources of income:

    a. earned:

        - salary (hourly, weekly, etc.),
        - professional fees,
        - self employment,
        - commission.

    b. other:

        - welfare,

- pensions,
- family allowance,
- interest,
- unemployment insurance,
- compensation.

41. To demonstrate an awareness of the major deductions which result in net income.

42. To demonstrate an awareness of the cost of living for:

    a. food,
    b. clothing,
    c. housing,
    d. transportation,
    e. recreation,
    f. insurance:

        - property,
        - automobile,
        - life,
        - crop,

    g. other.

43. To demonstrate the advantages, disadvantages, and the computations involving:

    a. cash vs. credit buying,
    b. borrowing money.

44. To distinguish between a census study and a sampling of the population.
45. To demonstrate an understanding of random samples and stratified samples.
46. To identify errors which can be made in sampling the population.

47. To collect data and to determine:

    a. mean,
    b. median,
    c. mode.

48. To set up frequency distribution tables.

49. To interpret the data illustrated in the following graphs:

    a. bar graph,
    b. histogram,
    c. line graph,
    d. frequency polygon,
    e. circle graph.

50. To illustrate data by drawing labelled graphs of the preceding types.
51. To identify congruent figures as having the same size and shape.
52. To relate congruence to the study of motion geometry and symmetry.
53. To use congruence notations.
54. To identify corresponding sides and corresponding angles of congruent figures.
55. To name polygons according to the number of sides.

56. To identify the properties of:

    a. triangles,
    b. quadrilaterals,
    c. other regular polygons.

57. To demonstrate an understanding of similarity as an enlargement or as a reduction of figures.
58. To use appropriate scale factors to enlarge or to reduce the size of an object.
59. To discover that similar figures have corresponding angles which are congruent.
60. To discover that similar figures have corresponding sides which are proportional (i.e., in the ratio given by the scale factor).

61. To discover the following by experimentation and direct measurement:

    a. the number of diagonals in a convex polygon,
    b. the angle sum for a polygon,
    c. the number of intersections of 2 lines, 3 lines, etc.
    d. in a circle, an angle at the centre is two times an angle at the circumference if they stand on the same arc,
    e. an angle on a semi-circle is a right angle,
    f. angles at the circumference subtended by the same

arc are congruent,
e. cyclic quadrilateral relationships.

62. To demonstrate an understanding of the concept that $m^0 = 1$ where $m \neq 0$.
63. To demonstrate an understanding of negative integral exponents.
64. To rewrite a given number in scientific notation and vice versa.
65. To multiply or divide two numbers using scientific notation.
66. To apply scientific notation and rounding off to estimate the product or quotient of very large or very small numbers.
67. To multiply a binomial by another binomial.
68. To divide a monomial by a monomial.
69. To divide a polynomial by a monomial.

70. To demonstrate an understanding of the terminology associated with coordinate graphing including:

    a. coordinate axes,
    b. quadrants,
    c. coordinates (ordered pairs),
    d. origin.

71. To plot a point given its coordinates and given a point to write its coordinates.
72. To generate ordered pairs given the values of "x" using the equation $y = ax + b$ where "a" and "b" may be any real numbers.
73. To graph linear equations in the form $y = ax + b$, using a table of values.
74. To rearrange any linear equation into the form $y = ax + b$.
75. To graph two linear equations, on the same graph, in order to determine the point of intersection.

Notes on Program Implementation

II. NOTES ON PROGRAM IMPLEMENTATION

A. Rationale

This guide contains the cognitive goals generally thought desirable for students at this time. The core objectives for each grade level are prescriptive but are not exhaustive. Teachers are encouraged to formulate learning outcomes for their own situation consistent with those in this guide. This guide has been established on the basis of the skills possessed by students graduating from elementary school and the needs which students have or will develop. For this reason, it is necessary for Grades 7-9 teachers to become familiar with the mathematics programs in the intermediate grades and in grade ten.

Students in grades 7-9 learn best through active personal involvement. Mathematics as a formal structure must be built upon a sound foundation of concrete experiences. According to developmental learning theory, students progress through three stages of learning: namely, concrete (objects), semi-concrete (pictures) and abstract (symbols). Care must be taken to articulate the Grades 7-9 program with patterns of development established in the K-6 guide where emphasis is placed upon the use of concrete materials. It may not be necessary or possible in every case to begin at the concrete level. If the material is of such a nature that first hand experience is impractical or if previous experiences have developed the underlying concepts, then initial activities may be at the semi-concrete or abstract levels. A student must be able to perform or "do mathematics". As students progress or as topics, concepts and processes require higher levels of cognitive development, the time devoted to concrete and semi-concrete activities decreases while the time spent in more abstract activities increases.

A balance should be achieved between basic skill development and an understanding of the structure and vocabulary of mathematics. To achieve this balance, terminology has been de-emphasized, understanding stressed, and appropriate practice recommended. It is essential that, once developed, each topic be reviewed and maintained.

Teachers are encouraged to employ a variety of teaching techniques incorporating visual aids, concrete materials, multi-texts, puzzles, games and activities. Some games, activities and suggested materials are included in this guide under "Notes to Teachers", and are marked by an asterisk (*). However, teachers are urged to develop their own aids to meet the needs of their students.

If the program is to be relevant to the present needs of students while also providing the foundations for future learning, then mathematics should be integrated with other subjects. A meaningful integration will go far towards achieving a deep and continuing interest in mathematics and an appreciation for its power and usefulness. The nature and means of integration will be determined at the school level by the specific programs being offered to the students. Teachers are encouraged to be aware of the ways of integrating or correlating mathematics with other subjects. Changes in the order of topics as presented in this guide may result.

B. Specific Goals of the Mathematics Program

In identifying the specific goals of the mathematics program, one must consider the present needs of individuals and society. The student goals upon which this guide is based are:

1. to increase understanding of the concepts and mastery of arithmetic skills;
2. to develop an understanding of the structure of the real number system;
3. to develop further understanding and facility with measurement including estimating and then verifying through the use of appropriate measuring instruments;
4. to observe and to identify geometric relationships involving both three-dimensional objects and two-dimensional figures;
5. to become more proficient in problem-solving;
6. to observe, to collect, to interpret and to display data graphically;
7. to realize the importance of mathematics for all consumers;
8. to read mathematics with understanding;
9. to cultivate positive attitudes toward, and a curiosity about, the world of mathematics;
10. to develop understanding and mastery of basic algebra skills.

C. The Guide

1. Description

The program outlined in this guide is a description of a set of basic skills for regular mathematics students. Work in subsequent grades relies on the mastery of the basic skills and the understanding of the concepts developed in this guide. Where necessary, students should have their program enriched.

Those sections of the guide dealing with grades 7 and 8 are designed to be taken in their entirety by regular students whereas the section dealing with grade 9 consists of two components; namely, core material and recommended options. The core material is a continuation of the grade 7 and 8 content, constitutes 70% of the grade 9 course, and is intended for all regular students. The remaining 30% of the time is to be devoted to the study of option topics. Three out of five topics are to be selected according to the interests and abilities of the students bearing in mind the course(s) for which students intend to register in grade 10.

2. Time Allotment

The essence of the Mathematics 7-9 guide is contained in the core objectives identified for each grade. It is recommended that on average, a minimum of 40 minutes per day or approximately 12% of the total time available, be allotted to the teaching of the program, although the needs of the particular students involved may require that varying amounts of time be spent to achieve the objectives. Whenever possible, teachers should include enrichment topics to enhance student learning experiences.

3. Unifying Themes

In order to maintain a balance in this program it was decided to identify four major skill areas; namely, arithmetic operations, estimation, problem solving and consumer mathematics applications, and measurement. Students are expected to master these skills. Teachers are encouraged to emphasize these skill areas at each and every opportunity. Where possible, each lesson and each topic should contain these skills. For this reason they have been called "unifying themes". These themes are of such importance that th , are sometimes included as subsections within major topics.

a) Arithmetic Operations - All students need to develop proficiency in recalling basic number facts, in using standard algorithms with reasonable speed and accuracy, and in performing mental calculations, as well as in developing an understanding of computational procedures. Students certainly need clear explanations of computational procedures and plenty of opportunity for drill and practice. For most grades 7-9 students there is, too often, nothing exciting about computation. The approach that a teacher uses is all-important in establishing a positive attitude toward computational activities. One way to catch the interest of students is to introduce them to new and relatively easy ideas from different areas of mathematics such as statistics or consumer mathematics. Other activities could include arithmetic contests, accuracy and speed tests, games, puzzles, the use of controlled readers and tachistoscopes, calculator drills and real-world problems. Teachers must strive for variety if students are to successfully learn arithmetic computation.

b) Estimation - One purpose of estimation is to provide a quick check on the reasonableness of answers in computation and problem solving. Estimation is a mental process, not a paper and pencil procedure. It is a difficult process for some students. One cause of this difficulty is the lack of prerequisite skills including the ability to round off numbers, proficiency in operating with multiples of 10, 100, 1000, etc., and facility with mental arithmetic. The responsibility for developing estimation skills must be shared among all mathematics teachers. Teachers must continually stress the use of rounding-off and estimation in arithmetic computation and problem solving.

Estimation is also an important aspect of measurement. It can be applied to a variety of numerical problems; for example, the number of people at a football game or the area of a floor to be covered with new carpet.

Students must be convinced that estimation should be used to save time and energy when precise calculations are not necessary.

c) Problem Solving and Consumer Mathematics Applications - The importance of problem solving in mathematics cannot be overemphasized. If the student knows immediately how to arrive at an answer to a problem, then the conscious problem solving process does not occur. The operations or manipulations involved are merely reinforcement of a mathematical skill. However, when hesitation occurs so that some deliberation is necessary before a method of solution is chosen, problem solving begins. Therefore, problem solving is the process involved whenever an individual encounters any situation where a desired solution is not immediately obvious. The problem may be very abstract as in pure mathematics, or it may be the application of a mathematical skill to a practical situation.

Suggested steps in problem solving include:

- understand the problem

  Encourage students to state the problem in their own words and determine answers to the following questions. What question does the problem ask? What is the estimated answer?

- devise a plan

  Possibly, students could draw a picture, make a graph or table, look for a pattern or write an equation as part of their plan for solving the problem. If all else fails students should try a completely new approach.

- carry out the plan

  Have students perform the steps in their plans by completing their diagrams, filling in their tables or solving their equations and then stating their conclusions.

- look back

  Students should always be certain that they have answered the question. Also, they should check the reasonableness of their answer, compare their calculated to their estimated answer, re-examine their solution for errors, and consider other ways to solve the problem.

The term "problem solving" should be interpreted in the broad sense. That is, teachers should not only consider word problems in the textbooks, but also classroom, community and social problems which require a solution. Mathematical problems are encountered in science, social studies, art, and other activities both in and out of school. Sample problems and applications must be drawn from the students' environment if students are to view mathematics as being relevant and practical for everyday life.

Proficiency in problem solving requires a positive attitude. This attitude may develop gradually as students gain confidence and mastery of their skills, learn the basic facts, learn to estimate quickly and learn to communicate mathematically. Although this positive attitude cannot be taught, teachers can play an important role in its development by encouraging discussions, initiating enjoyable activities using a variety of materials and accepting a diversity of approaches to the solution of a problem. To begin a success - enjoyment - interest pattern, teachers should choose problems which students can read, understand and solve. Once success in working problems has been established, the complexity of assigned problems should be gradually increased.

d) Measurement - Measurement in the grades 7-9 program is a continuation of the measurement strand from the elementary grades. As such, much of the work involves maintaining and extending skills which were introduced previously. The types of measurement to be reinforced include linear, area, volume, capacity, mass, temperature and time. Student activities should involve real life measuring experiences, estimations, and applying formulae. All measurements are to be done using SI units.

Teachers are reminded that estimation gives opportunity for students to be actively involved in their learning experiences. Activities need not be restricted to the classroom setting but should involve the totality of the students' environment.

Many measurement examples can be drawn from other subject areas. Science, physical education and outdoor education, to name a few, offer excellent opportunities for integration.

D. Role of Textbooks

Being independent of any textbook, this curriculum guide allows for greater flexibility in scope and approach than may be advocated by any particular author. Teachers are encouraged to plan their own programs using the guide as a framework. More than one book should be used in planning a program at a particular grade level. No one text should become the year's course. As most texts contain topics not included in the guide, teachers should not use a page-by-page approach.

It is exceedingly important that teachers have access to, and become acquainted with various resources, as this provides for flexibility in accommodating various ability and interest levels of individual students, variety of content and methodology, sources for exercises, activities and games to introduce and reinforce basic concepts, remediation and enrichment. Workbooks could be used for practice and consolidation of basic arithmetic operations but not for teaching.

E. Geometry

Geometry in the elementary program largely consists of recognizing attributes, ordering, and classifying objects in the real world and where possible, assigning quantitative or descriptive labels to these objects. The grades 7-9 program is an extension of this development. There is still a need for emphasis on manipulating three-dimensional objects and two-dimensional models.

Much of the study of plane geometry has traditionally involved formal terminology. Teachers are advised that the geometry sections in this guide do not stress definitions and theorems. Rather, emphasis is placed upon the relationship between figures, the properties of such figures and motion geometry involving flips, slides and turns.

Topics from geometry should be used at different times throughout the school year. Students should become actively involved in all aspects of geometry. Geometry activities could be viewed as a change of pace from arithmetic and algebraic operations.

F. Use of Calculators

The use of calculators is becoming common in schools. Teachers must accept the changes this will bring about in the field of mathematics. Time and accuracy are becoming increasingly critical in this age of automation and high labour costs. Therefore, teachers are encouraged to use the calculator to facilitate the learning process and to prepare students for a world of technology.

Basic concepts and computational skills must be learned and understood. Paper and pencil algorithms for these skills remain important in today's society. The main objectives in teaching need to be directed toward concept development, algorithmic processes and problem solving. Once these have been sufficiently learned, the calculator can be used to find solutions quickly and accurately.

For each mathematical problem, the calculator guarantees some answer. Hence, it is essential that students first estimate the answer to a problem, then calculate the solution, and then compare the two answers. Emphasis should be placed on estimation, on properly written solutions and on checking the reasonableness of answers.

In grades 7-9 mathematics classes, teachers could have calculators available for the purpose of encouraging discovery, exploration or creativity and for emphasizing more real-life problem solving and reinforcement.

Neither all schools nor all students can be expected to purchase calculators; therefore, their use must remain optional within the mathematics program at the present time.

G. Evaluation

Evaluation of student achievement and program effectiveness must be an integral, ongoing part of the teaching - learning process. "Any systematic attempt to develop basic skills must necessarily be concerned with evaluating and reporting pupil progress." (Arithmetic Teacher, October 1977). Evaluation should provide information regarding:

1. diagnosis and remediation. The purpose of diagnosis is to identify a student's strengths and weaknesses. In the case of weaknesses, the cause of the error may

then be identified and appropriate remediation provided by the teacher;

2. student growth in mathematical skills and understanding. This is an integral part of a developmental program. Because growth is continuous, evaluation must also be continuous. Teachers should assess a student's speed and accuracy, understanding of concepts and processes, ability in applying skills and should use evaluation to encourage further student attainment in mathematics;

3. learning strategies. Results should be used to judge the effectiveness of the instructional process - program of experiences, materials of instruction and strategies of teaching;

4. curriculum modification. Modifications should be ongoing. Therefore, continuing feedback from the schools regarding the appropriateness of the objectives and the degree of implementation of the current program is essential.

Testing is a vital component of evaluation. Tests may be structured to evaluate a specific concept, a series of interrelated concepts or all the concepts found in a year's work. Classroom teachers should design and use cumulative tests so that students maintain proficiency in the use of previously learned concepts and skills.

Students in grades 7-9 should be faced with some form of cumulative testing (be it cumulative unit tests or final examinations) involving the core objectives outlined in this guide. Both oral and written tests could be used. Every effort should be made to have such tests go beyond the lower level cognitive skills and to examine higher level cognitive objectives.

Evaluation is far more than paper and pencil testing. Teachers are reminded that evaluation of student achievement may take several additional forms such as daily observation, interviews or conferences with individual students, class discussions, and student reports and projects related to definitive and open-ended problem solving situations.

H. Consumer Mathematics

Consumer mathematics could be defined as mathematical applications pertaining to the wise use of money, statistical interpretation and measurement geometry. The purpose of consumer mathematics is to enable students to apply mathematical skills in a variety of real life, problem solving situations. It is suggested that consumer mathematics applications be integrated wherever possible into the mathematics program. The use of community resources such as field trips to local businesses, guest speakers and data from newspapers and other media are recommended.

In addition to the emphasis on practical or consumer mathematics, grades 7-9 students need to be made aware of the variety of career opportunities which require a mathematical background, as well as of the education or training that is prerequisite. Such topics as job applications, interview techniques, Social Insurance Numbers, and long-term job prospects could be discussed in conjunction with other subjects.

I. Organizing for Mathematics Instruction

The purposes of this guide are to identify the goals and objectives of the grades 7-9 mathematics program and to provide practical suggestions for teachers as they work with students to achieve these objectives. Although this guide outlines the concepts, skills and objectives, teachers are encouraged to plan their own course organization since each student has a different learning and maturation rate and, because of this, some students take longer than others to learn specific concepts and to develop certain skills.

It will be necessary to modify the program for educationally disadvantaged students. Suggestions and activities which may prove helpful to teachers of the educationally disadvantaged are found in a guideline entitled "Mathematics for Slower Learners, Grades 7-12" (1977 - Department of Education).

Enrichment materials should be provided for students whenever possible. It is generally hoped that enrichment materials will enhance and make the mathematics program more meaningful without significantly expanding into the mathematics content of the next grade. A list of such topics is found on pages 34 and 97 of the guide.

Scope and Sequence
Grades 6 - 9

UNIFYING THEMES

Concept or Operation	Grade 6	Grade 7	Grade 8	Grade 9
Whole Number Operations				
- addition, subtraction	- up to 6 digits	- any number of digits	M	M
- multiplication	- up to a combined total of 8 digits	- any number of digits	M	M
-division	- up to 6 digits by 3 digits	- any number of digits	M	M
Estimation	- multiplication, division	M	M	M
	- rounding off to hundredths	- rounding off to thousandths	M	M
	- measurement	M	M	M
Problem Solving and Consumer Mathematics	- stressed	- integrated core topic	M	M
		- introduce algebraic problem solving	- translate to equations and solve	M
				Option Topic Consumer Mathematics - income - net income - consumer expenditures - credit
Measurement				
- linear	- two place decimal fractions	M	M	M
		- perimeter of any polygon	M	M
			- circumference of circle	M
- area	- square and rectangle	M	M	M
		parallelogram, triangle	- circle	M
			- surface area of right circular cylinders	
- volume	- cubes using standard metric units		- concept of volume	
			- calculate volume of any rectangular solid	
			- problem solving	

M = Maintain and Develop

NUMBER SYSTEMS

Concept or Operation	Grade 6	Grade 7	Grade 8	Grade 9
Formal Properties		de-emphasize terminology	de-emphasize terminology	
	- commutative - associative - distributive			
	- Property of Zero	M	M	
	- Property of One	M	M	
		- closure	M	- summary
Natural Numbers	- identification	M	M	M
	- operations	M	M	M
Whole Numbers	- identification	M	M	M
	- operations	M	M	M
		- order of operations	M	M
		- powers	M	M
Integers			- identification	M
			- additive inverse	M
			- operations	M
				absolute value
Rational Numbers	- conversions	M	M	M
		- multiplicative inverse	M	M
			- definition	M
			- all operations	M
Irrational Numbers				- identification
Real Numbers				- definition
				- squares - square roots - Pythagorean relation - locating irrational numbers on a number line

M = Maintain and Develop

NUMBER THEORY

Concept or Operation	Grade 6	Grade 7	Grade 8	Grade 9
Even and Odd Numbers	- identification	M	M	M
Factors	- factors of a given number	M	M	M
	- common factors of two numbers	M	M	M
	- G.C.F. of two numbers	M	M	M
Multiples	- express multiples of a given number	M	M	M
	- find common multiples of two numbers	M	M	M
		- L.C.M. of two or more numbers	M	M
Primes and Composites	- identify primes and composites less than 100	M	M	M
	- factor trees	M		
		- write a composite number as the product of primes	M	M
		- relatively prime		
Divisibility Rules	- rules for divisibility by 2, 3, 4, 5, 6, 9, 10		M	
			- divisibility by 7, 8	
			- divisibility by composites	

M = Maintain and Develop

ALGEBRA

Concept or Operation	Grade 6	Grade 7	Grade 8	Grade 9
Open Number Phrases		- translation from word phrases to algebraic phrases	M	M
		- evaluate open number phrases in one variable	M	M
			- extend to two variables	M
				- classification
Equations	- open number sentences			
		- solve simple one variable equations by inspection		
			- introduce solving equations in one variable using equation solving techniques	M
				- variable on both sides of equal sign - fractional equations
Problem Solving		- introduction to algebraic problem solving	M	M
Operations			- addition and subtraction of like terms	M
				- multiplication of monomials - multiply a monomial and a polynominal

Option Topic
- binomial multiplication
- division by monomial
- exponents
  - zero exponent
  - negative exponents
  - scientific notation
  - Laws of Exponents

M = Maintain and Develop

FRACTIONS

Concept or Operation	Grade 6	Grade 7	Grade 8	Grade 9
Meaning	- part of whole - part of set - comparison (ratio) - division	M	M	M
Terminology	- numerator - denominator - proper fraction - improper fraction - mixed number - equivalent fraction	M	M	M
Equivalent Fractions	- concrete models	M		
	- generate sets of equivalent fractions	M	M	M
	- reduce fractions to lowest terms	M	M	M
Operations	- addition, subtraction	- treat mixed fractions as new work	M	M
	- multiplication: -fraction times whole number	M	M	M
		- proper fraction times proper fraction	M	M
		- proper fraction times mixed fraction	M	M
		- mixed fraction times mixed fraction	M	M
		- division: -proper fractions	M	M
		-mixed fractions	M	M

M = Maintain and Develop

DECIMAL FRACTIONS

Concept or Operation	Grade 6	Grade 7	Grade 8	Grade 9
Meaning	- read and write up to three decimal places	- at least 3 decimal places	M	M
		- expanded notation	M	
	- whole, tenths, hundredths	M	M	M
		- thousandths	M	M
Addition, Subtraction	- up to thousandths			
		- any number of places	M	M
	- horizontal and vertical notation	M	M	M
Multiplication	- decimal fraction times whole number	M	M	M
		- multiply by powers of 10	M	M
		- any decimal fraction times any other decimal fraction	M	M
Division	- up to 3-digit decimal divisor			
		- divide by powers of 10	M	M
		- decimal fraction divide by whole number	M	M
		- any decimal fraction divide by any other decimal fraction	M	M
Conversions	- express terminating decimal fractions up to 3 places in fractional notation	- convert any terminating decimal fraction to a fraction	M	M
		- convert any fraction to a decimal fraction	M	M
		- repeating decimal notation	M	M
			- convert simple repeating decimal fractions to fractions using "Pattern of Nines"	- convert any repeating decimal fraction into fraction

M = Maintain and Develop

RATIO, PROPORTION, PERCENT

Concept or Operation	Grade 6	Grade 7	Grade 8	Grade 9
Ratio	- notation	M	M	M
		- set up rates	M	M
Proportion	- as comparison of two equal ratios	- equivalent ratios	M	M
		- problem solving Scale drawings Map reading Unit pricing	M	M
				- similar triangles
Percent	- meaning	M	M	M
			- fractional percents	M
			- percents over 100	M
	- conversions (change percent to fraction or decimal fraction)	M	- all conversions	M
		- sales tax		
		- calculate specific percentage of a given number	M	M
		- calculate what percentage one number is of another	M	M
			- estimating percents	
			- calculate a number when a percentage of it is known	M
			- problem solving commission simple interest discount markup, markdown	M

M = Maintain and Develop

GEOMETRY

Concept or Operation	Grade 6	Grade 7	Grade 8	Grade 9
Lines and Line Segments		- estimate and measure	M	M
		- horizontal, vertical, oblique	M	M
		- parallel, perpendicular	M	M
	- symmetry	- lines of symmetry	M	M
Perimeter		- calculate for any polygon	M	M
Area		- square, rectangle, parallelogram	M	M
		- triangle		
Volume			- concept	
			- calculate volume for rectangular solids	M
Angles		- notation	M	M
	- classify	- acute, obtuse, right or straight	M	M
	- measure with protractor	- measure and draw any angles less than 180^{o}	- extend to 360^{o}	M
				- complementary, supplementary
				- opposite angles
				- corresponding angles
				- alternate angles
Triangles		- notation	M	M
	- classification by sides	M		
		- classification by angles		
		- lines of symmetry		
		- area		

Concept or Operation	Grade 6	Grade 7	Grade 8	Grade 9
Motion Geometry	- slides, flips, turns, combinations	M	M	
		- rotational symmetry	M - parallel lines - perpendicular lines - properties of motions	
Quadrilaterals	- square, rectangle, parallelogram		M - kite - lines of symmetry - rotational symmetry	M - identify properties
Circles	- terminology		M - arc, sector - determine circumference - determine area - surface area of right circular cylinders	
	- interpret circle graphs		M - construct circle graphs	
Construction Geometry				- basic compass and straightedge constructions - special angles - copy triangles - designs
Congruence				- notation - relate to motion geometry and symmetry - corresponding angles and sides

GEOMETRY - Page 3

Concept or Operation	Grade 6	Grade 7	Grade 8	Grade 9
Similarity				- enlargement or reduction - scale factors - corresponding angles - corresponding sides
Inductive Geometry				- discover relationships by direct measurement

M = Maintain and Develop

GRAPHING

Concept or Operation	Grade 6	Grade 7	Grade 8	Grade 9
Pictograph	- interpret - construct	M M		M M
Bar Graph (vertical or horizontal)	- interpret - construct	M M		M M - histogram
Line Graph	- interpret - construct	M M		M M - frequency polygon
Coordinate Graph	- locate points on the graph	- locate points and identify coordinate pairs in the first quadrant - picture plotting - enlargement or reduction	- expand to all four quadrants	M Option Topic - graph equations of the form y=ax+b - graphically determine the point of intersection for two straight lines
Circle Graph	- interpret only		- interpret - construct	

M = Maintain and Develop

STATISTICS

Concept or Operation	Grade 6	Grade 7	Grade 8	Grade 9
Sampling				Option Topic - census - sampling random stratified
Averages	- arithmetic mean		M - median	- errors in sampling M M - mode
Frequency Tables				- class boundaries, tally, frequency - mean
Interpreting Data	- bar graph	M		M - histogram
	- line graph	M		M - frequency polygon
	- circle graph		M	
Illustrating Data	- bar graph	M		M - histogram
	- line graph	M		M - frequency polygon
			- circle graph	M

M = Maintain and Develop

Grade 7

GRADE 7 MATHEMATICS GUIDE

The following mathematics guide lists the core objectives for regular grade 7 students. Teachers are reminded that these objectives constitute a minimum program and that enrichment topics should be included, whenever possible.

Enrichment topics could include: optical illusions, perspective drawings, calculators and computers, orienteering, puzzles and problem solving, preparation for mathematics contests, mathematics applications in the community, historical topics, women in mathematics, measuring instruments, alternate computation methods, taxicab geometry, sequences and series, logic activities, topology, colour theorems, nomographs, solid geometry, surveying, paper folding or mathematical experiments.

INDEX TO TOPICS

I. WHOLE NUMBER SYSTEM

1. Historical Development
2. Natural Numbers
3. Whole Numbers
4. Place Value
5. Rounding Off
6. Basic Operations
7. Order of Operations
8. Exponents
9. Problem Solving

II. ALGEBRA

1. Open Number Phrases
2. Equations
3. Problem Solving

III. NUMBER THEORY

IV. FRACTIONS

1. Meaning of a Fraction
2. Types of Fractions
3. Equivalent Fractions
4. Operations with Fractions
5. Problem Solving

V. DECIMAL FRACTIONS

1. Concept
2. Expanded Notation
3. Rounding Off
4. Conversions
5. Operations with Decimal Fractions
6. Problem Solving

VI. RATIO AND PROPORTION

1. Ratio and Rates
2. Proportion

VII. PERCENT

1. Meaning of Percent
2. Conversions
3. Problems and Applications

VIII. GEOMETRY AND MEASUREMENT

1. Lines and Line Segments
2. Perimeter
3. Area
4. Angles
5. Triangles
6. Motion (Transformational) Geometry

IX. GRAPHING

1. Data Graphing
2. Coordinate Graphing

CORE OBJECTIVES	NOTES TO TEACHERS
	NOTE: See the section entitled "Unifying Themes" on pages 18-19. The skills identified therein are of such importance that teachers must emphasize their use at every opportunity.
I. WHOLE NUMBER SYSTEM	
1. Historical Development	
The student should be able to discuss the relative efficiency of the Hindu-Arabic number system with at least one other number system.	The extent of the comparison will depend upon student interest. Possible systems to consider are the Babylonian, Roman, Mayan, or Egyptian. Stress should be placed on reading mathematics as well as on library research. Students could develop their own number system using some of the properties of the system(s) studied.
2. Natural Numbers	
The student should be able:	
a) to identify the set of natural or counting numbers (N);	Students should be aware that the set of natural or counting numbers is an infinite set. At this stage the concept and notation of infinite sets should be discussed. $N = \{1, 2, 3, \ldots\}$
b) to apply the Property of One for Multiplication.	Students are not to memorize a symbolism for the Property of One for Multiplication (i.e., PIM $\forall_a$ $a \in N$, $a \times 1 = a$). Rather, students are to understand that multiplying any natural number by "one" leaves the number unchanged.
3. Whole Numbers	
The student should be able:	
a) to illustrate the set of whole numbers (W) as an extension of the set of natural numbers (N);	$W = \{0, 1, 2, \ldots\}$
b) to apply the Property of Zero for Addition and Multiplication;	Students are to understand that adding zero to any number leaves the number unchanged and that multiplying by zero results in zero for an answer. Students are not required to memorize PZA: $\forall_a$ $a \in W$, $a + 0 = a$ nor PZM: $\forall_a$ $a \in W$, $a \times 0 = 0$
c) to illustrate the Closure Property as it applies to whole numbers.	This is new work for grade 7 students. Stress that a set is said to be "closed" under an operation if the answer is in the set after performing the operation on any two numbers in the set.

CORE OBJECTIVES

NOTES TO TEACHERS

Example:

The set of whole numbers is not closed under subtraction since, for example, 5 - 7 is not a whole number.

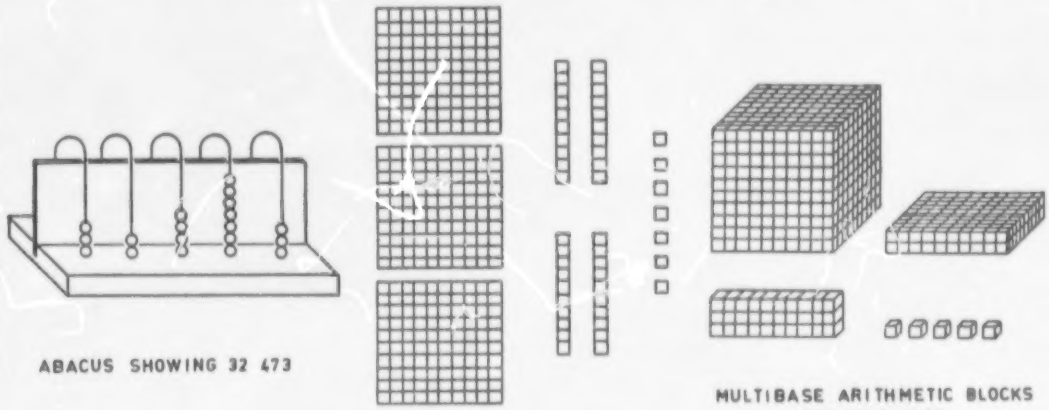
Sets of numbers other than the whole numbers could be tested for closure (even, odd, Fibonacci, etc.).

4. Place Value

The student should be able:

a) to explain and to illustrate the structure of the Hindu-Arabic or decimal numeration system;

Place value materials provide a concrete representation of our base ten system and also of systems in other bases. Materials for this purpose include place value cards, multibase arithmetic blocks, the abacus, and place value pocket charts. A student may be asked to represent a given number or to state a number represented by a collection of material. The following figures show numbers represented by various kinds of place value materials.



ABACUS SHOWING 32 473

PLACE VALUE CARDS SHOWING 347

MULTIBASE ARITHMETIC BLOCKS SHOWING 1235

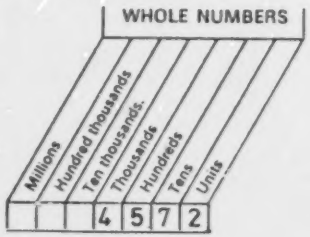
(courtesy Silver Burdett Company)

* Suggested materials: abacus, Dienes blocks, Multibase arithmetic blocks, (MAB blocks), graph paper, Cuisenaire rods.

CORE OBJECTIVES

b) to rewrite numbers in expanded format, without using exponents, and vice versa.

NOTES TO TEACHERS

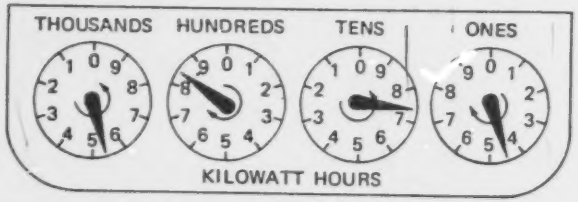


The place value of a digit in a numeral is either 10 times the place value of the digit to its right or $\frac{1}{10}$ times the place value of the digit to its left. In the number 4572, the place value of 7 is tens. This is 10 times the place value of the 2 and $\frac{1}{10}$ times the place value of the 5.

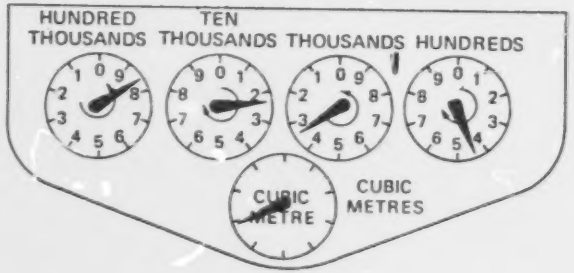
Bases other than ten could be used to emphasize the structure of the base 10 number.

$375 = 300 + 70 + 5$
$= (3 \times 100) + (7 \times 10) + (5 \times 1)$

Examples of expanded notation from a student's environment could include reading meters for electricity and natural gas.



On this hydro-electric meter, the last dial on the right records kilowatt hours from 0 to 10. The other dials record the number of tens, hundreds, and thousands of kilowatt hours used. The reading on the meter is 5874 kilowatt hours.



Natural gas is another energy source that is commonly used. Gas use is measured in units of 100 m^3 (cubic metres). The figure shows the dials on a common type of gas meter. The upper four dials are used for reading the meter. The lower dial is used only to test the accuracy of the meter. Each mark on the dial at the far right records up to 10 x 100 m^3, or 1000 m^3. The reading on the meter is 8234 x 100 m^3 or 823 400 m^3.

CORE OBJECTIVES	NOTES TO TEACHERS
5. Rounding Off The student should be able to round off whole numbers.	Rounding off is used primarily for the process of estimation. Rounding off and estimation activities go hand in hand and should be a part of daily classroom activities. Emphasize that the basic value of the number must be maintained; 5764 rounded off to the nearest thousand is 6000 not 6. Also, "5" or over is rounded up. 649 to the nearest hundred is 600; 650 to the nearest hundred is 700; and 651 to the nearest hundred is 700.
6. Basic Operations The student should be able to estimate, to calculate and to check the solution to any problem involving the operations of addition, subtraction, multiplication and division of whole numbers.	It is essential that students achieve competence in basic operations. See note on "Arithmetic Operations" on page 18. Estimation and checking should be stressed as essential steps in all calculations. Students must be asked to estimate answers. This does not imply that a student first calculate the answer and then round off. Rather, the student must round off the numbers under consideration, perform the calculation mentally, record the estimated answer, compute the actual answer and then compare the estimated and actual answers. * Suggested materials: magic squares, cross number puzzles, Lattice Multiplication, Casting Out Nines, Napier's Bones, calculators, Games - Tuf, Heads Up, Ordo, Match the Product, Magic Stars.
7. Order of Operations The student should be able to estimate and to calculate the solution to number phrases involving several operations with or without brackets.	This is a new topic for grade 7 students and the need for an "order of operations" should be established. Questions from several sources should be used here. Priority rule order: 1. Operations within brackets (parentheses) first. 2. Operations of multiplication and division. 3. Operations of addition and subtraction. 4. Operations of the same priority are performed left to right. Examples: 1. $2 + 3 \times 5 = 2 + 15 = 17$ (multiply first). 2. $12 \div 3 \times 4 = 4 \times 4 = 16$ (equal priority so operations are performed left to right). 3. $(2 + 4) \times 5 = 6 \times 5 = 30$ (operations in brackets performed first). Teachers should emphasize that in a series of products the distributive property does not apply.

CORE OBJECTIVES

NOTES TO TEACHERS

Example:

2 x 3 x 4 does not equal (2 x 3) x (2 x 4).

Also, students must realize that 67 + 92 x 0 is not equivalent to zero.

NOTE: Students are to be familiar with the commutative, associative and distributive properties only in order to understand their usefulness in estimating and simplifying operations. Emphasis is not to be placed on formal terminology. For example, students are not to memorize a + (b + c) = (a + b) + c as being the associative property for addition. Rather, they are to concentrate on identifying 68 + (32 + 47) as being equivalent to (68 + 32) + 47 in order to facilitate mental arithmetic calculations. Other examples might include:

1. 74 x 15 = 74 x (10 + 5)
   = (74 x 10) + (74 x 5)
2. 68 x 99 = 68 x (100 - 1)
   = (68 x 100) - (68 x 1)
3. 8 x 79 x 125 = (8 x 125) x 79
   = 1000 x 79

8. Exponents

The student should be able:

a) to demonstrate an understanding of the meaning of "power", "base", and "exponent";

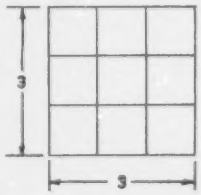
"2^3" is a power with "2" as the base and "3" as the exponent.

b) to convert a power to a simple number and vice versa;

Greek mathematicians often represented numbers with dots arranged in geometric shapes. This is how square numbers got their name.

Also, it is possible to draw diagrams to represent square numbers. The sides of the square represent the two factors.

$3^2 = 3 \times 3 = 9$



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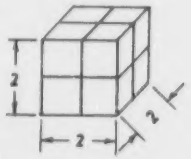
CORE OBJECTIVES

NOTES TO TEACHERS

c) to apply the concept of exponents to expanded notation;

Similarly with cubes, the length, width and height could represent the three identical factors.

$2^3 = 2 \times 2 \times 2 = 8$



NOTE: Teachers are encouraged to emphasize concrete models when introducing powers. However, it is expected that students will achieve facility with the operations outlined in this objective.

Previous work: $375 = 300 + 70 + 5$
$= (3 \times 100) + (7 \times 10) + (5 \times 1)$

New work: $375 = (3 \times 10^2) + (7 \times 10^1) + (5 \times 1)$

d) to perform simple calculations involving powers using the order of operations.

NOTE: With the introduction of powers it is necessary to establish a new priority rule order:

1. Operations within brackets (parentheses) first.
2. Operations with powers.
3. Operations of multiplication and division.
4. Operations of addition and subtraction.
5. Operations of the same priority are performed left to right

$3 \times 2^4 = 3 \times (2 \times 2 \times 2 \times 2) = 3 \times 16 = 48$

* Suggested materials: Cuisenaire rods, cubes, decoder puzzles.

9. Problem Solving

The student should be able to solve and to check word problems pertaining to any of the preceding concepts or skills which involve whole numbers.

See note on problem solving on page 19.

Problems could be drawn from mathematical experiments (those experiments which use some degree of computation in relating mathematics to other disciplines such as physics, psychology or biology). For example, compare breathing rates under specified conditions, for different ages or even for different species. The average rate for an adult at rest is from 14 to 18 breaths per minute.

Consumer education is another excellent source of problems involving operations with whole numbers. Basic arithmetic operations are necessary for completing profit and loss statements, inventory cards, single-page cashbooks and charts for showing gross income per period or credit extension.

CORE OBJECTIVES

NOTES TO TEACHERS

Example 1:

What is the next number in the following sequence?

24 12 36 24 48 36 ____

Solution:

The sequence follows either of the patterns below:

a) - 12 + 24 - 12 + 24 - 12 + 24 so the next number is 60.
b) - 12 x 3 - 12 x 2 - 12 x 1 so the next number is 36.

NOTE: The operations above indicate how to determine the next number in the sequence from the previous one.

Example 2:

A certain banker tests applicants for the position of teller by posing this problem: "Here is a cheque for $63.00. Give me that amount in six bills, but do not give me any $1.00 bills." If the applicant cannot solve this problem in 30 seconds he doesn't get the job. What is the solution?

Solution:

1 - $50.00 bill; 1 - $5.00 bill; 4 - $2.00 bills.

Example 3:

A student had 47 gerbils. All but 9 of them died. How many were left?

Answer:

9 left.

Example 4:

John is five years older than Pete. Tilly is two years younger than John. Bob is Tilly's twin brother. If John is 28 years old, how old are the others? Is Bob an identical twin?

Solution:

Pete is 23; Bob and Tilly are 26.
Bob is not an identical twin if Tilly is female and Bob is male.

CORE OBJECTIVES	NOTES TO TEACHERS
	Example 5: Under normal conditions, your heart beats 72 times per minute. Emory's heart beats 90 times every minute when he is running and 100 times a minute when he is running up stairs. If Emory rests for two minutes and then runs for five minutes, how many times will his heart beat? Who is Emory running from? Is Emory's heartbeat the same as yours? Solution: Assumptions are inherent in all problem solving. Discuss the assumptions which one could make in solving this problem. Emory's heart beats (2 x 72) + (5 x 90) = 144 + 450 or 594 beats. Encourage students to create unusual answers to this question. Emory's heartbeat is not the same as mine but you might have a chance!
II. ALGEBRA 1. Open Number Phrases The student should be able: a) to express word phrases as number phrases or expressions and vice versa;	A distinction between "open" (contain a variable) and "closed" number phrases should be made. Considerable emphasis is to be placed on this topic. Examples and exercises from more than one source will need to be used.
b) to evaluate open number phrases by substitution.	Students are expected to master this topic. Care must be taken to choose examples which are within the present level of student development. For example, questions involving negative numbers should not be used at this time. Students should be encouraged to use equal signs correctly. A useful format is given in the following example. Example: Evaluate 3x + 5 when x = 14 Solution: $3x + 5 = 3(14) + 5$ $= 42 + 5$ $= 47$

CORE OBJECTIVES

NOTES TO TEACHERS

2. Equations

The student should be able to solve simple one variable equations.

Students should be encouraged to obtain the solution by inspection or by using the so-called "cover-up" technique. If equation properties are to be introduced at this point, considerable time should be spent emphasizing the balance in an equation. At this time all equations should have whole number solutions.

Students in grade 7 are to represent the unknown quantity by some variable, say "n", rather than using a box or blank.

Example:

Solve for n if 2n + 3 = 11. Check your answer.

Solution:

2n + 3 = 11 (cover the "2n")
2n = 8 (cover only the "n")
n = 4

Check by substitution. Three possible formats have been included.

2n + 3 = 11	2n + 3 | 11	Use L.S. for "left side" and R.S.
2(4) + 3 = 11	2(4) + 3 | 11	for "right side".
8 + 3 = 11	8 + 3 | 11	L.S. = 2(4) + 3
11 = 11	11 | 11	= 8 + 3
		= 11
		R.S. = 11
		So L.S. = R.S.

Solution is n = 4

* Suggested materials: Games - Heads Up, Tuf, Guess My Rule, Equation Rummy.

3. Problem Solving

The student should be able to translate simple word problems to equations and then to solve the equations.

See note on problem solving on page 19.

Students have difficulty understanding the need for detailed written solutions to problems if the answers are obvious. It is suggested that for problems involving very simple equations, such as n + 3 = 8, students should be required only to translate from the written problem to the appropriate algebraic symbols. This could be viewed as a reading or translating exercise.

CORE OBJECTIVES

NOTES TO TEACHERS

Example:

Determine the equation for the following word problem but do not solve.

"A student eats a slice of white bread and a slice of brown bread for lunch. If the student receives 119 calories from this lunch and a slice of white bread contains 62 calories, how many calories are contained in a slice of brown bread?"

Possible Solution:

Let "n" be the number of calories in a slice of brown bread.
$62 + n = 119$

In more difficult problems, students should be required to give complete algebraic solutions and should be encouraged to seek alternate methods of solution. Students should be given some credit for correct non-algebraic methods of solution to any problem. After all, the strategies of problem solving are a much more worthy goal than the solution of dozens of type problems.

Example 1:

During a tic-tack-toe tournament, Judy won three times as many games as Rolfe. Judy won 105 games; how many did Rolfe win?

Solution:

Let "n" be the number of games which Rolfe won.
$3n = 105$ (cover-up technique)
$n = 35$

Check:

$3n = 105$
$3(35) = 105$
$105 = 105$
Rolfe won 35 games.

Example 2:

The population of a certain city is now 48 000. If the population increases by 200 people each year, in how many years will the population reach 53 000?

CORE OBJECTIVES

NOTES TO TEACHERS

Solution:

Let "n" represent the number of years

$$48\,000 + 200n = 53\,000$$
$$200n = 5000 \text{ (cover-up technique)}$$
$$n = 25$$

(Encourage estimation here. Difficulties arise at this stage due to students' inability to multiply and divide by powers of 10).

Check:

$$48\,000 + 200n = 53\,000$$
$$48\,000 + 200(25) = 53\,000$$
$$48\,000 + 5000 = 53\,000$$
$$53\,000 = 53\,000$$

Population will reach 53 000 in 25 years.

NOTE: Students should be encouraged to create their own word problem exercises.

III. NUMBER THEORY

The student should be able:

a) to list the set of factors of a given number;

Stress an understanding of the term "factor(s)".

Factors may be determined by:

1. using division or multiplication;
2. making a graph paper chart as illustrated on the next page.

CORE OBJECTIVES

b) to list the set of multiples of a given number;

c) to determine the greatest common factor (GCF) of a set of composite numbers;

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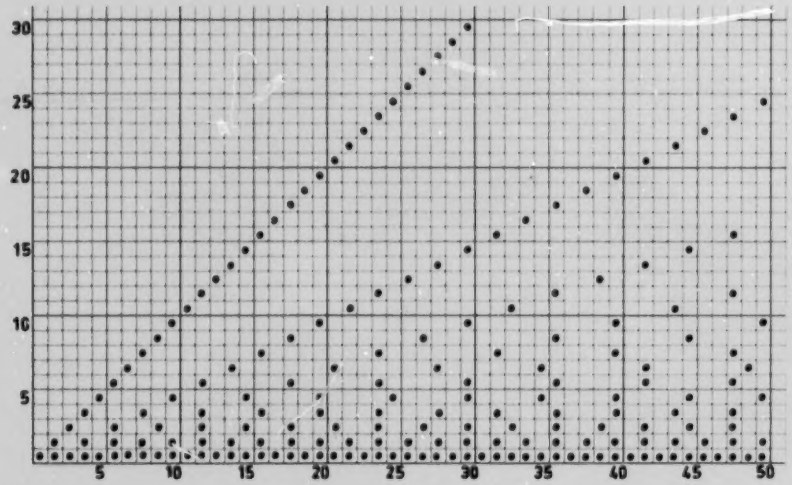
NOTES TO TEACHERS



Number the axes as shown. On the first row, place a dot in every square. On the second row, place a dot in every second square. On the third row place a dot in every third square, etc.

The chart can be used for finding factors, LCM's and GCF's. To find the factors of say, 24, locate 24 on the horizontal axis. Each dot above 24 signifies one of its factors. The value of these factors is read off the vertical scale (i.e., 1, 2, 3, 4, 6, 8, 12, 24).

This is a most important concept to develop in preparation for reduction of fractions and for finding a common denominator.

Example: Find the GCF of 16 and 24

Solution 1

Factors of 16 are 1, 2, 4, 8, 16.
Factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24.
Common factors are 1, 2, 4, 8.
GCF is 8.

Solution 2

To use the previous chart to find the GCF of say, 16 and 24, locate the two numbers on the horizontal axis. The factors are located above each number. The highest dots which line up horizontally are the value of the GCF. In this case, 8 is the GCF.

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CORE OBJECTIVES

NOTES TO TEACHERS

d) to determine the least common multiple (LCM) of a set of numbers

Example: Find the LCM of 6 and 8.

Solution 1

Multiples of 6 are 6, 12, 18, 24, 30, 36, 42, 48,...
Multiples of 8 are 8, 16, 24, 32, 40, 48,...
Common multiples of 6 and 8 are 24, 48.
LCM is 24.

Solution 2

To use the chart to find the LCM of say, 6 and 8, choose 6 and 8 on the vertical axis. Move horizontally along the rows opposite 6 and 8. Locate the first time where two dots line up vertically. The value of the LCM is the corresponding value on the horizontal axis. In this case, 24 is the LCM.

e) to explain and to give examples of prime and composite numbers;

When considering the natural numbers the number "1" is neither prime nor composite.

Discuss with students various methods of finding prime numbers on a grid. Seek the most efficient method.

The method used by Eratosthenes was:

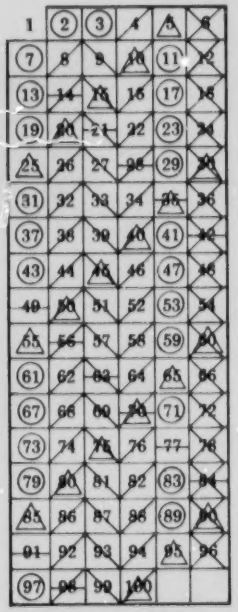
1. Draw a diagonal (/) through all the multiples of 2, except 2. Note the pattern.
2. Draw a diagonal (\) through all the multiples of 3, except 3. Note the pattern.
3. Draw a triangle (△), or use a coloured marker, over all the multiples of 5, except 5. Note the pattern.
4. Draw a line (—), or use a coloured marker, through all the multiples of 7, except 7. Note the pattern.

Is it necessary to proceed with numbers larger than 7 when considering a hundred grid?

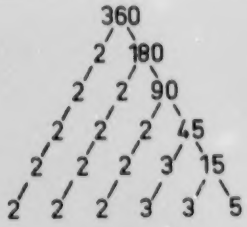
The remaining (circled) numbers are prime. Note the pattern.

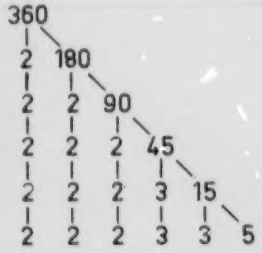
If students use a 10 by 10 array the pattern in the arrangement of the primes is not as regular as the one shown in the diagram.

For additional information on "Number Theory" see the K-6 Mathematics Guide, pages 243-251.



CORE OBJECTIVES	NOTES TO TEACHERS
f) to write a composite number as a product of prime numbers;	A factor tree could be used to determine prime factors.





$360 = 2 \times 2 \times 2 \times 3 \times 3 \times 5$ (prime factors)

$360 = 2^3 \times 3^2 \times 5$ (exponential notation)

g) to identify and to give examples of numbers that are relatively prime.	Relatively prime numbers are numbers which have a GCF of 1.
	* Suggested materials: Games - Prime Factor Rummy, Prime Factors Game.
IV. FRACTIONS	Common fractions have much less prominence in SI than in the Imperial system of measurement. However, fractions will be in everyday use for some time to come. No matter how small a unit is chosen, that unit will not measure each physical situation a whole number of times.
	For years fractions have been taught before decimals. However, the present emphasis on the metric system with its characteristic decimal properties makes an earlier treatment of decimal fractions worthwhile. In many ways operations with decimal fractions are more closely related to operations with whole numbers than with fractions. Also, current research seems to indicate merit in teaching decimal fractions before common fractions. Teachers are encouraged to make their own decision in this matter. If decimal fractions are to be taught before common fractions teachers are advised that some methodology suggested under "Decimal Fractions" would need to be changed.
	NOTE: Classes should not spend long periods of time doing only fraction exercises. It is suggested that geometry and graphing exercises be interspersed with the work on fractions.

CORE OBJECTIVES

NOTES TO TEACHERS

1. Meaning of a Fraction

The student should be able:

a) to demonstrate an understanding of a fraction as:

i. a part of a whole,
ii. a part of a set,
iii. a comparison (ratio),
iv. a division;

Although most of the material presented in this section should be familiar to the students, it is important that students are certain of the terminology and concepts. Research has shown that when dealing with fractions, both the active use of manipulative materials by students and teacher demonstration of concepts using materials are superior to methods in which materials are not used.

Examples:

- part of a whole:

The shaded portion depicts $\frac{1}{3}$ of the whole.

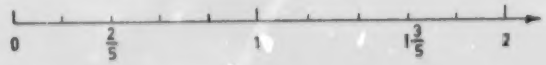
- part of a set: examples could be created using sets such as a deck of cards, letters of the alphabet, a set of teeth, etc.
- ratio: examples could involve probability and odds, scale diagrams, similar triangles, recipes, etc.
- division: a fraction may imply the operation of division. For example, $\frac{2}{3}$ is a way of writing $2 \div 3$.

b) to identify the numerator and denominator of a fraction;

* Suggested materials: Dienes blocks, MAB blocks, fraction charts, geoboards, fraction discs, coloured cubes, fraction squares, fraction rectangles, Games - Fraction Snap, Beat My Fraction, Fraction Dominoes, Fraction War, Snakes and Ladders (fractions), Fraction Race Game, Happy Pairs.

Fractions should be written with a horizontal line. For example, write $\frac{3}{4}$ not 3/4 and $3\frac{1}{4}$ not $3\frac{1}{4}$ or $\frac{1}{34}$.

c) to locate fractions on a number line and to identify the fractional equivalent of a point on a number line.



2. Types of Fractions

The student should be able:

a) to identify proper and improper fractions;

CORE OBJECTIVES

b) to identify mixed fractions or mixed numbers.

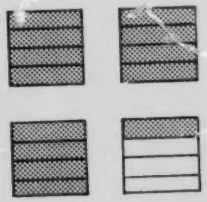
3. Equivalent Fractions

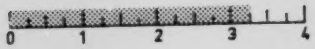
The student should be able:

a) to identify equivalent fractions;

NOTES TO TEACHERS

Each of the following depicts $3\frac{1}{4}$.

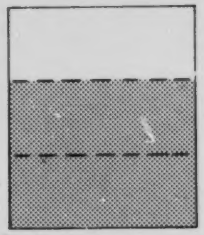


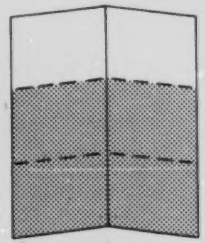


(courtesy Silver Burdett Company)

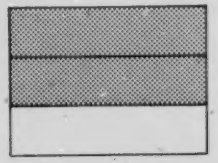
Material representations showing equivalent fractions could include:

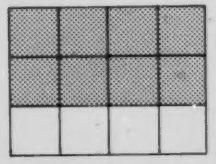
1. paper folding -





2. geometric figures -





This illustration could be performed on a geoboard or on graph paper.

CORE OBJECTIVES

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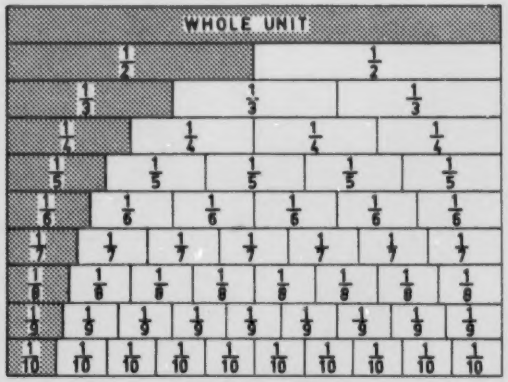
3. fraction chart -



b) to generate sets of equivalent fractions;

The concept of equivalent fractions should be developed with concrete materials prior to using the Property of One. A thorough understanding of the concept of equivalent fractions is essential.

$$\frac{2}{3} = \frac{2}{3} \times 1 = \frac{2}{3}$$

$$\frac{2}{3} = \frac{2}{3} \times \frac{2}{2} = \frac{4}{6}$$

$$\frac{2}{3} = \frac{2}{3} \times \frac{3}{3} = \frac{6}{9}$$

$$4 = \frac{4}{1} \times \frac{3}{3} = \frac{12}{3}$$

c) to change improper fractions to mixed fractions and vice versa;

Example:

$$2\frac{1}{4} = 2 + \frac{1}{4}$$

$$= \frac{8}{4} + \frac{1}{4}$$

$$= \frac{9}{4}$$

Ultimately establish the algorithm that $2\frac{1}{4} = \frac{(2 \times 4) + 1}{4} = \frac{9}{4}$.

CORE OBJECTIVES

d) to reduce fractions to lowest terms;

NOTES TO TEACHERS

If students understand the concept of equivalent fractions they should have little difficulty in reducing fractions.

After considerable work, students should be able to reduce fractions using multiplication by a form of 1. Students should ultimately learn to look for the greatest common factor of the numerator and the denominator.

NOTE: Changing $\frac{6}{5}$ to $1\frac{1}{5}$ is not considered reducing but rather converting to a mixed fraction.

Example:

Reduce $\frac{10}{15}$ to lowest terms.

Solution:

$$\frac{10}{15} = \frac{2 \times 5}{3 \times 5}$$

$$= \frac{2}{3} \times \frac{5}{5}$$

$$= \frac{2}{3} \times 1$$

$$= \frac{2}{3}.$$

Students should understand this algorithm before other methods of reducing fractions are used. Teachers are advised to delay introducing cancellation techniques until at least grade 8.

Another useful method for reducing fractions is based on the following algorithm. Find the difference between the denominator and the numerator of the fraction to be reduced. Either this difference or a factor of this difference will be the greatest common divisor of the fraction.

Example 1:

Reduce $\frac{110}{121}$.

CORE OBJECTIVES

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Solution:

Denominator - numerator or 121 - 110
= 11 (Since "11" is a prime number the greatest common divisor in this example will be 11 or 1).
11 divides into 121 and 11 divides into 110.

$$\frac{110}{121} = \frac{10 \times 11}{11 \times 11}$$

$$= \frac{10}{11}.$$

Example 2:

Reduce $\frac{28}{63}$.

Solution:

Denominator - numerator or 63 - 28
= 35
Since 35 does not divide evenly into 28 then a factor of 35 is the greatest common divisor in this example. Factors of 35 are 1, 5 or 7. The correct choice is 7.

$$\frac{28}{63} = \frac{4 \times 7}{9 \times 7}$$

$$= \frac{4}{9}.$$

e) to compare fractions.

Convert each fraction to equivalent fractions having the same denominator. Then, numerators can be compared easily.

Example:

Is $\frac{4}{5}$ greater than, less than, or equal to $\frac{5}{8}$?

Solution

$$\frac{4}{5} = \frac{4}{5} \times \frac{8}{8} = \frac{32}{40}$$

$$\frac{5}{8} = \frac{5}{8} \times \frac{5}{5} = \frac{25}{40}$$

$$\frac{32}{40} > \frac{25}{40} \text{ then } \frac{4}{5} > \frac{5}{8}.$$

CORE OBJECTIVES

NOTES TO TEACHERS

4. Operations with Fractions

The student should be able:

a) to add and to subtract:

i. proper fractions,

Students should be able to generalize this procedure to arranging a series of fractions in ascending or descending order.

Teachers could encourage students to use their knowledge of LCM to assist in comparing fractions.

All answers should be expressed in lowest terms. The fractions involved in these operations should be limited to those in common usage such as halves, thirds, quarters, fifths, sixths, eights and tenths.

Methods used for the addition or subtraction of fractions must emphasize equivalent fraction techniques. Avoid algorithms which employ cross-multiplication in any form.

A high level of competency should be attained in the addition and subtraction of fractions. A maintenance program should be developed in order to retain this proficiency.

A student's image of fractions is not easy to determine but it is thought that "two thirds add three eights" creates an image of five something, and that there is a real possibility that such a false image creates confusion when he/she sees the following:

$$\frac{2}{3} + \frac{3}{8} = \frac{25}{24}.$$

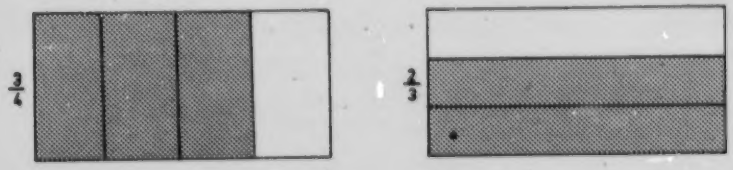
The following example will show how to use materials in order to subtract fractions.

Example:

Compute $\frac{3}{4} - \frac{2}{3}$.

Solution:

Make a rectangle to represent each fraction as shown.



CORE OBJECTIVES

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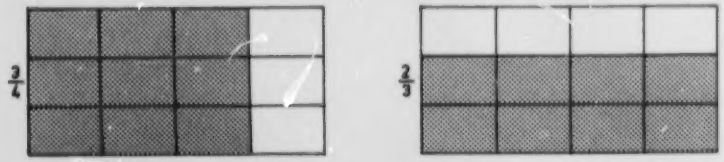
Note that one rectangle is divided vertically, the other horizontally. It is necessary to remove the amount of shaded area in the second from the amount of shaded area in the first. Thus, the first rectangle must be divided horizontally, in thirds, and the second rectangle, vertically, in quarters, as shown.



There are now 12 smaller regions in each of the original rectangles. The 8 shaded regions comprising the $\frac{2}{3}$ model must be removed from the shaded region in the $\frac{3}{4}$ model. This leaves one shaded region which is $\frac{1}{12}$ of the original rectangle. Hence, $\frac{3}{4} - \frac{2}{3}$ becomes $\frac{9}{12} - \frac{8}{12} = \frac{1}{12}$.

If additional suggestions regarding the use of concrete materials and other methodologies for the addition and subtraction of fractions are required, see the K-6 Mathematics Guide, pages 158, 159, 215-217 and 277-281.

* Suggested materials: Games - Ordo, Fraction Action.

ii. mixed fractions;

While many students may have been introduced to the following concepts, this section should be treated as new work.

Example 1:

Find the value of $6 - 2\frac{2}{3}$.

Solution:

$$6 - 2\frac{2}{3} = 5\frac{3}{3} - 2\frac{2}{3}$$
$$= 3\frac{1}{3}.$$

CORE OBJECTIVES

NOTES TO TEACHERS

Example 2: Find $6\frac{2}{3} + 4\frac{3}{4}$.

Solution 1

$$6\frac{2}{3} + 4\frac{3}{4} = 6\frac{8}{12} + 4\frac{9}{12}$$
$$= 10\frac{17}{12}$$
$$= 10 + 1\frac{5}{12}$$
$$= 11\frac{5}{12}$$

Solution 2

$$6\frac{2}{3} = 6\frac{8}{12}$$
$$+ \ 4\frac{3}{4} = 4\frac{9}{12}$$
$$= 10\frac{17}{12}$$
$$= 10 + 1\frac{5}{12}$$
$$= 11\frac{5}{12}$$

Example 3: Find $4\frac{1}{6} - 2\frac{2}{3}$.

Solution 1

$$4\frac{1}{6} - 2\frac{2}{3} = 4\frac{1}{6} - 2\frac{4}{6}$$
$$= 3 + \frac{6}{6} + \frac{1}{6} - 2\frac{4}{6}$$
$$= 3 + \frac{7}{6} - 2\frac{4}{6}$$
$$= 1\frac{3}{6}$$
$$= 1\frac{1}{2}$$

Solution 2

$$4\frac{1}{6} = 3\frac{7}{6}$$
$$- \ 2\frac{2}{3} = 2\frac{4}{6}$$
$$= 1\frac{3}{6}$$
$$= 1\frac{1}{2}$$

Stress an understanding of regrouping or of "carrying" and "borrowing" in questions involving the operations of addition and subtraction of mixed fractions.

Some students may use methods which involve improper fractions. There are advantages to such methods, but the numerators tend to become large.

b) to multiply:

When multiplying, students are often confused with the false equivalence that "to multiply" means "to increase". This could create problems when multiplying either fractions or decimal fractions.

Multiplication of fractions should be treated as a new topic.

CORE OBJECTIVES

i. proper fraction times whole number,

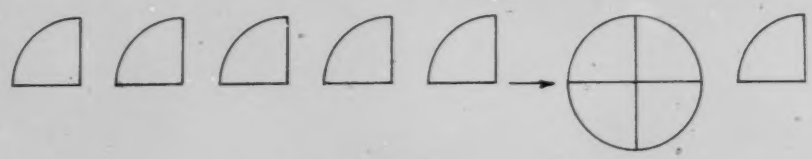
ii. mixed fraction times whole number,

iii. proper fraction times proper fraction,

iv. proper fraction times mixed fraction,

NOTES TO TEACHERS

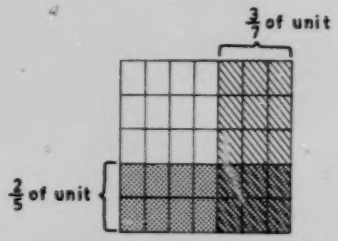
The following examples show that multiplication is equivalent to repeated addition.



$$5 \times \frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{5}{4} = 1\frac{1}{4}$$

Students, when using the algorithm, must realize that $5 \times \frac{1}{4}$ is equivalent to $\frac{5}{1} \times \frac{1}{4}$.

Examples in which there is no reducing are clearest for showing the algorithm for multiplying two proper fractions. The diagram shows that $\frac{2}{5}$ of $\frac{3}{7}$ is $\frac{6}{35}$.



The Multiplication Algorithm is $\frac{2}{5} \times \frac{3}{7} = \frac{2 \times 3}{5 \times 7} = \frac{6}{35}$.

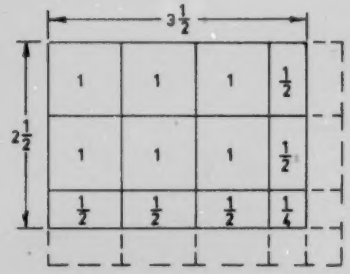
Example:

Find $\frac{2}{3} \times 1\frac{4}{5}$.

Solution:

$$\frac{2}{3} \times 1\frac{4}{5} = \frac{2}{3} \times \frac{9}{5}$$

$$= \frac{2 \times 9}{3 \times 5} = \frac{18}{15} = 1\frac{3}{15} = 1\frac{1}{5}.$$

CORE OBJECTIVES	NOTES TO TEACHERS
v. mixed fraction times mixed fraction;	Students should realize that $2\frac{1}{2} \times 3\frac{1}{2} \neq (2 \times 3) + (\frac{1}{2} \times \frac{1}{2})$ or $6\frac{1}{4}$ This may be demonstrated by the diagram on the right.  OR $2\frac{1}{2} \times 3\frac{1}{2} = \frac{5}{2} \times \frac{7}{2}$ $= \frac{35}{4}$ $= 8\frac{3}{4}$
	Note of Caution: Cancelling techniques should not be employed at this stage of a student's development.
c) to find the multiplicative inverse or reciprocal of any fraction or whole number;	Stress is to be placed on the fact that the product of a number and its reciprocal is "one". NOTE: 0 has no unique reciprocal.
d) to divide:	Teachers are advised that just as some students form a false equivalence between "multiply" and "increase" some will also form a similar false equivalence between "division" and "decrease". NOTE: Stress that division is the inverse of multiplication. The models which can be used for division of fractions include sets, regions and repeated subtraction. These are effective in explaining only very simple problems and in giving the students an intuitive grasp of the division of fractions. sets: If you have 3 dozen eggs and use $\frac{1}{2}$ dozen each morning, how many mornings will the eggs last? Essentially this question asks, how many $\frac{1}{2}$'s are in 3? Draw a figure and count the number of half dozens. The conclusion is that $3 \div \frac{1}{2} = 6$. regions: Betty made a pan of fudge. She will sell $\frac{1}{2}$ of the fudge at the school bake sale in packages which hold $\frac{1}{8}$ of the pan. How many packages will she have for sale? Essentially this question asks, "how many $\frac{1}{8}$'s are there in $\frac{1}{2}$"? The diagram on the following page shows that $\frac{1}{2} \div \frac{1}{8} = 4$.

CORE OBJECTIVES

NOTES TO TEACHERS

i. proper fractions,

FUDGE FOR SALE

repeated subtraction: The previous two models suggest repeated subtraction. How many times can $\frac{1}{2}$ be subtracted from 3?

3	$2\frac{1}{2}$	2	$1\frac{1}{2}$	1	$\frac{1}{2}$
$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$	$-\frac{1}{2}$
$2\frac{1}{2}$	2	$1\frac{1}{2}$	1	$\frac{1}{2}$	0

The conclusion is that $3 \div \frac{1}{2} = 6$.

Several algorithms could be used for division of fractions:

1. "Definition of Division" Algorithm

Consider the following intuitive approach:

$8 \div 4 =$
$8 \times \frac{1}{4} =$

$12 \div 6 =$
$12 \times \frac{1}{6} =$

$25 \div 5 =$
$25 \times \frac{1}{5} =$

The answer in each group is the same. What has happened? (Division has been replaced with multiplication by the reciprocal.) Can this always be used?

By the definition of division $\frac{2}{5} \div \frac{3}{4} = m$ is equivalent to $m \times \frac{3}{4} = \frac{2}{5}$.

CORE OBJECTIVES	NOTES TO TEACHERS
	$\frac{2}{5} \div \frac{3}{4} = \frac{2}{5} \times \frac{4}{3}$ since division by a quantity $(\frac{3}{4})$ is equivalent to multiplication by the reciprocal of that quantity $(\frac{4}{3})$. Hence, $\frac{2}{5} \div \frac{3}{4} = \frac{2}{5} \times \frac{4}{3}$ Stress that division is the inverse of multiplication.
ii. mixed fractions.	Once the students have established the algorithm for division of proper fractions (i.e., division by a quantity is equivalent to multiplication by the reciprocal of that quantity) it can then be extended to mixed fractions.

Example 1:

$$6 \div 2\frac{1}{3} = \frac{6}{1} \div \frac{7}{3}$$
$$= \frac{6}{1} \times \frac{3}{7}$$
$$= \frac{18}{7}$$
$$= 2\frac{4}{7}$$

Example 2:

$$5\frac{1}{4} \div 3\frac{1}{2} = \frac{21}{4} \div \frac{7}{2}$$
$$= \frac{21}{4} \times \frac{2}{7}$$
$$= \frac{42}{28}$$
$$= \frac{3}{2} \text{ (This may require more than one step).}$$
$$= 1\frac{1}{2}$$

CORE OBJECTIVES

NOTES TO TEACHERS

2. Complex Fraction Algorithm

Example:

$$\frac{2}{5} \div \frac{3}{4} = \frac{\frac{2}{5}}{\frac{3}{4}}$$

$$= \frac{\frac{2}{5}}{\frac{3}{4}} \times 1$$

$$= \frac{\frac{2}{5}}{\frac{3}{4}} \times \frac{\frac{4}{3}}{\frac{4}{3}}$$

$$= \frac{\frac{2}{5} \times \frac{4}{3}}{\frac{3}{4} \times \frac{4}{3}}$$

$$= \frac{2}{5} \times \frac{4}{3}$$

$$= \frac{8}{15}$$

Hence, $\frac{2}{5} \div \frac{3}{4} = \frac{8}{15}$.

3. Common Denominator Algorithm

Example:

$$\frac{2}{5} \div \frac{3}{4} = \frac{8}{20} \div \frac{15}{20}$$

$$= \frac{8 \div 15}{20 \div 20}$$

$$= 8 \div 15$$

$$= \frac{8}{15}$$

CORE OBJECTIVES

NOTES TO TEACHERS

5. Problem Solving

   The student should be able to solve and to check word problems involving fractions.

The following method employs the property of one and techniques similar to those employed in the common denominator algorithm.

Example:

$$\frac{2}{5} \div \frac{3}{4} = \frac{3 \times 2}{3 \times 5} \div \frac{3}{4} \text{ (Introduce } \frac{3}{3} \text{ since 3 is found in the divisor).}$$

$$= \frac{4 \times 3 \times 2}{4 \times 3 \times 5} \div \frac{3}{4} \text{ (Introduce } \frac{4}{4} \text{ since 4 is found in the divisor).}$$

$$= \frac{4 \times 3 \times 2 \div 3}{4 \times 3 \times 5 \div 4}$$

$$= \frac{4 \times 2}{3 \times 5}$$

$$= \frac{8}{15}$$

It is expected that students would ultimately combine the first two steps.

Vocational mathematics textbooks could be used as one source for problems involving fractions.

Example:

In a certain machine shop 1320 parts must be manufactured. If $\frac{3}{4}$ of the job has been done, how many parts have been made?

Solution:

Number made = total number x fraction of job done

$$\text{Number made} = 1320 \times \frac{3}{4}$$

$$= \frac{1320}{1} \times \frac{3}{4}$$

$$= \frac{3960}{4}$$

$$= 990$$

990 parts have been made.

CORE OBJECTIVES

NOTES TO TEACHERS

V. DECIMAL FRACTIONS

Present trends in our society are causing the earlier introduction and emphasis of decimal fractions. The use of the metric system, based on subdivisions and multiples of 10, along with the increased use of calculators, in which both input and output are in decimal form, has led to these trends. For these reasons students are required to master the work on decimal fractions.

Work on decimal fractions should be presented in conjunction with place value, our monetary system and the metric system. Suggestions include:

1. examine the structure of these systems;
2. perform operations using metric and monetary units;
3. convert metric units to other metric units using the operations of multiplication and division with powers of 10.

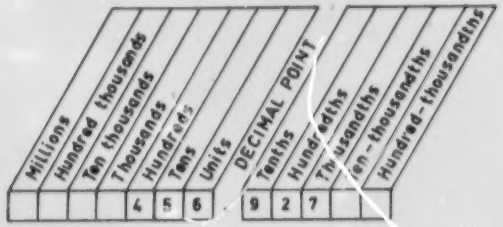
The word "decimal" is derived from the Latin "decimus", which means one-tenth.

1. Concept

The student should be able:

a) to read decimal fractions to at least three decimal places;

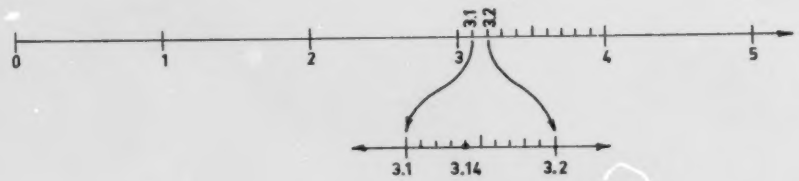
Review place value as needed.



When having students read decimal fractions, reserve the word "and" to separate whole numbers from the decimal portion. For example, read 456.927 as four hundred fifty-six and nine hundred twenty-seven thousandths.

If no natural number precedes a decimal point, place a zero in front of the decimal point. This is SI notation.

Students need to know that a whole number can be written with a decimal point. Otherwise, confusion may arise in problems such as 128 + 5.6.

CORE OBJECTIVES	NOTES TO TEACHERS
b) to locate decimal fractions on a number line.	As long as a student continues to study mathematics, the number line and its extensions will be used. It is advantageous for students to plot points on a number line magnified to show tenths or hundredths as this helps students to see the order as well as the density of points. Example: Place 3.14 on a number line. 
2. Expanded Notation The student should be able: a) to write a decimal fraction in expanded notation without exponents; b) to write a decimal fraction in expanded notation using only positive exponents.	Refer to the place value chart on page 63. $456.927 = (4 \times 100) + (5 \times 10) + (6 \times 1) + \left(9 \times \frac{1}{10}\right) + \left(2 \times \frac{1}{100}\right) + \left(7 \times \frac{1}{1000}\right)$ $456.927 = (4 \times 10^2) + (5 \times 10) + (6 \times 1) + \left(9 \times \frac{1}{10}\right) + \left(2 \times \frac{1}{10^2}\right) + \left(7 \times \frac{1}{10^3}\right)$
3. Rounding Off The student should be able: a) to round off decimal fractions to the nearest whole number;	Rounding off is to be used in conjunction with estimation. Students should round off and estimate answers at every opportunity. Remember "5" or over is rounded up. 76.49 to the nearest whole number is 76. Only the "4" is to be considered. Do not round off from the right. 76.5 to the nearest whole number is 77.

CORE OBJECTIVES	NOTES TO TEACHERS
b) to round off decimal fractions to the nearest tenth, hundredth or thousandth.	43.647 to the nearest tenth is 43.6.
4. Conversions	
The student should be able:	
a) to change a fraction to a decimal fraction;	Students should realize that any fraction converts, upon division, to either a terminating or a repeating decimal fraction. Students should be aware of the various notations for repeating decimals. For example, $\frac{1}{9} = 0.111\ldots = 0.\overline{1} = 0.\dot{1}$ or $\frac{2}{7} = 0.\overline{285714} = 0.\dot{2}85714\dot{}$.
b) to rewrite a terminating decimal fraction as its fractional equivalent.	Use could be made of the Property of One for Multiplication. Example: Convert 0.43 to a fraction. Solution: As 0.43 is read as "43 hundredths" write the answer as $\frac{43}{100}$. Students in grade 7 are not required to convert repeating decimal fractions to fractions. * Suggested materials: MAB blocks, 100 grid paper, graph paper, geoboards, hundred tray, Game - Decimal Hopscotch.
5. Operations with Decimal Fractions	Teachers are reminded that the following methodology employs knowledge of fractions. If decimal fractions are to be taught prior to fractions, different methodologies must be developed.
The student should be able:	
a) to add and to subtract decimal fractions with any number of decimal places;	The validity of operations with decimal fractions may be verified using fractional equivalents. Measurement in the metric system and decimal representation of money could also be used to explain the addition or subtraction of decimal fractions. The computational procedures for decimal fractions parallel the procedures for whole numbers. Decimal place values can be represented on an abacus, a place value chart or a place value grid. The algorithms for addition and subtraction of decimal fractions differ from the corresponding algorithms for whole numbers in that the decimal points are aligned but the right hand digit may not align. In order to add or subtract digits that have the same place value the decimal points must align.

CORE OBJECTIVES	NOTES TO TEACHERS
	Use the column form for adding and subtracting decimal fractions. Encourage the use of "0" as a placeholder in questions of the following type. Example: Find the value of 7.6 - 6.84. Solution: $\begin{array}{r} 7.60 \\ -6.84 \\ \hline 0.76 \end{array}$
b) to multiply or to divide a decimal fraction by a power of ten (i.e., 10, 100, 1000, etc.);	Students need to be proficient in multiplying and dividing whole numbers by powers of 10 before attempting the work with decimal fractions. Proficiency in both multiplication and division of decimal fractions by powers of ten are necessary if students are to become accurate and relatively quick in estimating answers to mathematical problems. The following example outlines one possible explanation. Example: What is the value of 0.71 x 1000? Solution: $0.71 \times 1000 = \frac{71}{100} \times 1000$ $= \frac{71\ 000}{100}$ $= 710$ It is hoped that students will become proficient in using the appropriate algorithm (multiplying by 1000 moves the decimal point three places to the right, etc. Some teachers may prefer to say that it is the digits rather than the decimal point which move since the place value of the digits change). The multiplication of decimal fractions using concrete materials provides opportunities to review and to build understanding of the place value system and the operation of multiplication itself. Example: Compute 0.6 ÷ 10.

CORE OBJECTIVES

NOTES TO TEACHERS

Solution:

$$0.6 \div 10 = \frac{6}{10} \div 10$$
$$= \frac{6}{10} \times \frac{1}{10}$$
$$= \frac{6}{100}$$
$$= 0.06$$

See the section entitled Operations with Fractions on page 54 for other methods.

It is hoped that students will become proficient at moving the decimal point when multiplying or dividing by powers of ten.

c) to multiply a decimal fraction by a whole number other than a power of 10;

In order to show concretely the multiplication of 3 and 0.7 one could use three rectangular regions divided into tenths. Shade $\frac{7}{10}$ of each figure and add to get the answer (repeated addition method).

$= \frac{21}{10} = 2\frac{1}{10} = 2.1$

OR $0.7 + 0.7 + 0.7 = 2.1$

OR $3 \times 0.7 = 3 \times \frac{7}{10}$

$= \frac{21}{10}$

$= 2\frac{1}{10}$

$= 2.1$

A number line could also be used.

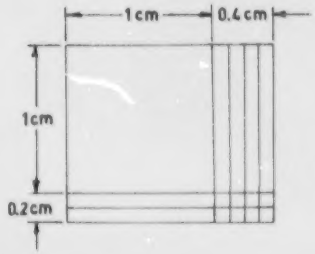
CORE OBJECTIVES

NOTES TO TEACHERS

d) to multiply a decimal fraction by a decimal fraction with any number of decimal places;

The area of a rectangle can be used to show that the multiplication of tenths times tenths yields hundredths as the answer. The following diagram shows a rectangle which is 1.4 cm by 1.2 cm. The product of 1.4 cm and 1.2 cm is 1 square cm plus 6 strips each of $\frac{1}{10}$ of a square cm plus 8 small squares each $\frac{1}{100}$ of a square cm.

1.4 cm x 1.2 cm = 1 square cm + $\frac{6}{10}$ square cm + $\frac{8}{100}$ square cm = 1.68 square cm.



Any problem in multiplication of decimal fractions may be done by multiplying whole numbers and then placing the decimal point appropriately. If students prematurely know the rule for placing the decimal point there is little motivation for thinking carefully about the problem using concrete materials.

Establish an algorithm (rule) for multiplication: the product has as many decimal places as the sum of the decimal places in the factors.

Students are required to become proficient in the multiplication of decimals. Mastery is expected.

e) to divide a decimal fraction by a power of ten;

Proficiency in both multiplication and division of decimal fractions by powers of ten are necessary if students are to become accurate and relatively quick in estimating answers to mathematical problems. Students need proficiency in dividing whole numbers by powers of ten before the work with decimals is begun.

f) to divide a decimal fraction by a whole number other than a power of ten;

The concept of finding the average or arithmetic mean of a series of numbers can be used as a source of questions for this objective.

NOTE: The introduction of division of decimal fractions in grade 7 is a good time for review of the process of division and affords students the opportunity to extend their understanding of the decimal numeration system.

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CORE OBJECTIVES	NOTES TO TEACHERS
g) to divide a decimal fraction by a decimal fraction involving any number of decimal places.	Discuss the need for division of decimals. Applied problems involving metric measures or money can be used to give meaning to division of decimals. For example, the following problem can be used to reinforce the presentation of division as repeated subtraction. Example: A carpenter cuts shelves each 1.2 metres long from a board 5.3 metres long. How many shelves can he get from one board? The procedure for moving decimal points can be justified and explained by the familiar operations on fractions. $0.2\overline{)0.18}$ or $\frac{0.18}{0.2} = \frac{0.18 \times 10}{0.2 \times 10} = \frac{1.8}{2}$ or $2\overline{)1.8}$ NOTE: The answer for $0.2\overline{)0.18}$ is the same as the answer for $2\overline{)1.8}$. It is important that students realize that the question was changed only in order to make the division easier. This, in effect, establishes the division algorithm for decimal fractions. Teachers must realize that students have difficulty in the correct placement of the decimal point in the solution of both multiplication and division questions involving decimal fractions. The most common error is in the placement of the decimal point. Students should be encouraged to check answers by multiplication using estimation.
6. Problem Solving The student should be able to solve and to check word problems involving decimal fractions.	See notes on Problem Solving on page 19. Teachers are encouraged to pay particular attention to the relevance of sample problems involving decimal fractions. Metric measurement activities, olympic records and problems from industrial mathematics could be used here. The following are examples of such problems. Solutions have not been included. Examples: 1. The diameter of a tennis ball is 6.35 centimetres. The diameter of a golf ball is 4.13 centimetres. How many times wider is a tennis ball than a golf ball? 2. C. Barnett of Canada at one time held the world record for roller skating. She skated 32.124 kilometres in one hour. What was her rate in metres per minute? 3. If a power drill cuts into a certain metal at the rate of 0.6 mm per revolution, how many revolutions are needed to drill a hole 6.24 mm deep?

CORE OBJECTIVES

NOTES TO TEACHERS

Additional problems can be found in consumer and business mathematics exercises involving budgets, cashbooks, cheques and cheque stubs, utility bills, chequing accounts, perimeter and area.

References: (available from Department of Education)

1. Idea Bank of Consumer Education Learning Activities for K to 6
2. Consumer Education Learning Modules for the Middle Years
3. Consumer Education Resource Material Grades 7-12

Sample Problem:

The school yearbook committee received $2800.00 in student fees and $3685.00 from advertisers last year. The committee spent $4925.00 for publishing; $278.00 for photography supplies; $115.00 for the editor's seminar; $383.00 for promotion; $560.00 for the marketing class which sold advertising for the committee; and $107.17 for postage and stationery. Prepare a profit and loss statement for the committee for the school year ending June 30 of last year.

School Yearbook Committee
Profit and Loss Statement
for the period September 1, 19 ___ to June 30, 19 ___

DATE	DESCRIPTION	INCOME	EXPENSES	BALANCE
	Student fees	2800.00		
	Advertising	3685.00		
	Publishing		4925.00	
	Photography supplies		278.00	
	Editor's seminar		115.00	
	Promotion		383.00	
	Marketing class expenses		560.00	
	Postage and stationery		107.17	
	TOTAL	6485.00	6368.17	116.83

NOTE: Teachers may prefer to introduce some geometry at this time. The sections entitled "Lines and Line Segments" and "Perimeter and Area" provide a change of pace and applications for the work on decimal fractions.

CORE OBJECTIVES	NOTES TO TEACHERS
VI. RATIO AND PROPORTION	
1. Ratio and Rates	
The student should be able:	This is a new topic for many grade 7 students. Piaget's research has shown that students are unable to deal successfully with this topic until they reach the stage of formal operations which occurs around age 12 or 13. For this reason, teachers are advised to introduce this section of the course by using examples from the students' environment. Students should be encouraged to find examples such as anti-freeze solutions, frozen orange juice mixtures, map scales, scale drawings and blueprints, pricing practices, gear ratio, compression ratio, automobile gas consumption, concrete mixtures, pitch of roofs, batting averages, recipes, etc.
a) to demonstrate an understanding of ratio as a comparison of two numbers, using the notation a to b or a:b or $\frac{a}{b}$;	Students should realize that there are two main ways to compare numbers; one is by subtraction, while the other is by division (ratio). Ratios, like fractions, are to be expressed in simplest or reduced form. For example, the ratio 125:1000 is usually written 1:8.
b) to set up a ratio between two quantities having the same unit of measure;	Examples could be drawn from measurement geometry, statistics (ratio of teachers to students in your school), science (introduction to physics in grade 7 science guide), etc.
c) to set up a rate between two quantities having different units of measure.	Students should also realize that rates can be worked as ratios. In this case comparison is made with completely different units. Examples could include pricing (2 pencils for 25¢), reading rates (pages or words per minute), automobile gas consumption (kilometres per litre), batting averages expressed as decimals, and probabilities or odds. * Suggested materials: hundred cube tray, graph paper, measuring tapes, scales.
2. Proportion	
The student should be able:	
a) to demonstrate an understanding of a proportion as equivalent ratios;	Consumer examples are most useful for introducing this concept. Introductory discussion might be centered around newspaper advertisements. Introduce the term "equivalent ratios" and the term "proportion" to describe an equation of equivalent ratios. Only integral values are to be used. Students are encouraged to establish proportions to solve problems.

CORE OBJECTIVES

b) to apply and to solve problems relating to:

i. scale drawings,

NOTES TO TEACHERS

Scale drawings are useful in many areas. Some drawings are larger than the true objects, while others are smaller.

NOTE: In printing this guide, pages were reduced in dimension. Therefore, measurements referring to the size of diagrams originally drawn to scale are not accurate.

Example 1:

Find the true length in millimetres of the following ladybug which is drawn 4 times its normal size.

Ladybug (×4)

Solution:

The sketch measures 19 mm.
Hence, the true length is $19 \div 4 = 4.75$ mm or 5 mm rounded off to the nearest millimetre.

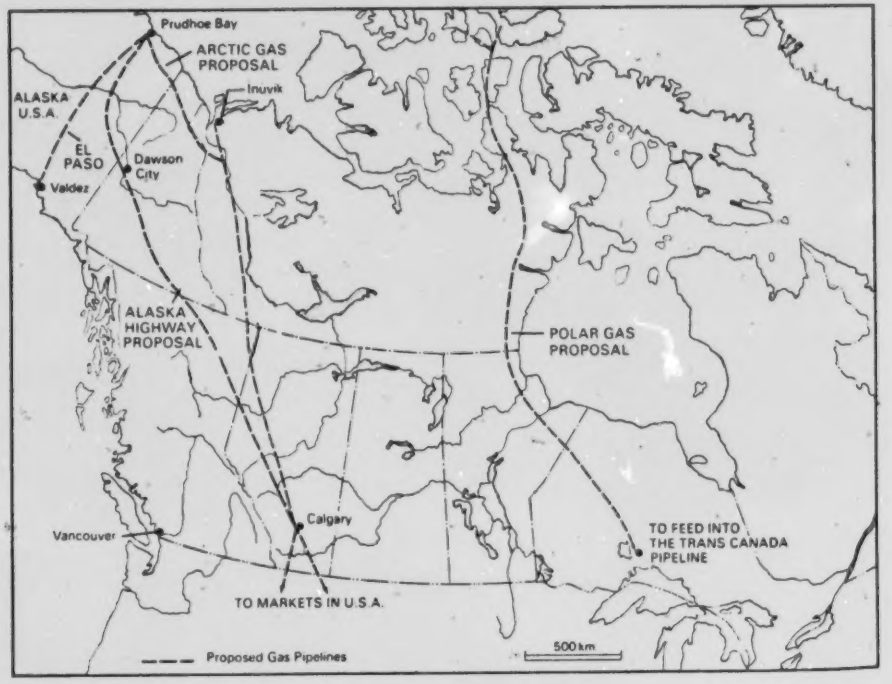
Also, a proportion could be set up, $\frac{19}{4} = \frac{n}{1}$, and solved.

Example 2:

Use the following scale drawing of a drama stage to suggest problems for students.

Students should also be given opportunities to make their own scale drawings given a particular scale. A good suggestion is for students to start with a door lock plate.

CORE OBJECTIVES	NOTES TO TEACHERS
ii. map reading,	Students are usually interested in examining and/or creating street or road maps of their own locale. This activity can involve scale drawings and map reading. It is to be noted that most maps found in local municipal offices are expressed using Imperial measure. However, when using maps from textbooks use only those which have metric scales. Recent social studies texts contain maps similar to the following. Students could be asked to determine the length of the proposed polar gas line, the shortest distance from Winnipeg to Calgary, etc.
iii. unit pricing,	Real-life situations should be an integral part of the grade 7 mathematics program. Occasionally, real-life problems, less structured than those found in textbooks, should be presented orally. These need not require exact answers. Textbooks, periodicals, newspapers, magazines and everyday experiences are all good sources. Students could be encouraged to visit their local supermarket to determine data useful in unit pricing.



(courtesy McGraw-Hill Ryerson)

CORE OBJECTIVES	NOTES TO TEACHERS
iv. miscellaneous problems.	Example: Consider the following data on dishwasher detergents: Brand A: 992 grams for $1.95. Brand B: 1.42 kilograms for $2.59. Brand C: 1.84 kilograms for $3.39. Which is the better buy? Why? Students doing questions of this type are more easily motivated if allowed to use calculators to perform the division operations. Example: Last season the Jets won 3 out of every 5 hockey games they played. How many games did the team win in its 80 game regular season schedule? Solution: Let the number of games won be "n" Then, $\frac{3}{5} = \frac{n}{80}$ $\frac{3}{5} \times 1 = \frac{n}{80}$ $\frac{3}{5} \times \frac{16}{16} = \frac{n}{80}$ $\frac{48}{80} = \frac{n}{80}$ $48 = n$ The Jets won 48 hockey games.
VII. PERCENT 1. Meaning of Percent The student should be able to demonstrate an understanding of percent as a special ratio of a number compared to 100.	Students in grade 7 may not have been exposed to the concept of percent in elementary school. For this reason, it should be stressed that percent is just a ratio with a denominator of 100. Encourage students to identify various familiar uses of percent. These may include antifreeze solutions, sales tax, commissions, salary increases, Gallup poll data, etc.

CORE OBJECTIVES

NOTES TO TEACHERS

In order to introduce percent using a hands on approach, give students a 10 by 10 grid. Have them shade, say 30% red and 20% green. Ask questions such as: "What percent is unshaded?" "What is the total percent shaded?", etc.

Some educators prefer to define 1% as $\frac{1}{100}$. Thus, $3\% = 3 \times \frac{1}{100} = \frac{3}{100}$. This method may be extended to work with percentages over 100 and fractional percentages when the need arises.

Percents containing fractions (i.e., $12\frac{1}{2}\%$) should not be considered here.

Students must realize that 100% represents the totality under consideration.

2. Conversions

The student should be able:

a) to change fractions to percent and vice versa;

Teachers could use the conversions in this section to strengthen and maintain previous work on proportions and on changing from fractions to decimals.

Example:

Convert $\frac{2}{5}$ to its equivalent percent.

1. Proportion method:

Set up the proportion $\frac{2}{5} = \frac{n}{100}$ and solve for "n"

$$\frac{2}{5} = \frac{40}{100}$$

$$\frac{2}{5} = 40\%$$

2. Arithmetic method:

$\frac{2}{5} = \frac{2}{5} \times 1$ (Property of One for Multiplication)

$= \frac{2}{5} \times 100\%$

$= 40\%$

b) to change decimal fractions to percent and vice versa.

Students should memorize decimal fraction and percent equivalents of common fractions: halves, thirds, quarters, fifths and tenths. For example, $\frac{1}{4} = \frac{25}{100}$ or 25% or 0.25.

CORE OBJECTIVES

3. Problems and Applications

The student should be able:

a) to calculate a specific percentage of a given number;

NOTES TO TEACHERS

Students are expected to be able to convert percents to decimal fractions by dividing mentally using 100.

Percent without applications is of little value. Applications must be an integral part of teaching percent.

There are three distinct methods of solving percent problems. However, the proportion method could be considered preferable because it can be used for all types of percent problems, it uses equivalent ratios, and it makes sense because it is based on the meaning of a percent.

Example:

If the local baseball team won 40% of the 25 games played how many games did the team win?

1. Proportion method (uses equivalent fractions to solve):

Let "n" be the number of games won.

Because $40\% = \frac{40}{100}$

Then $\frac{40}{100} = \frac{n}{25}$

$n = 10$

The team won 10 games.

2. Percent to a decimal fraction or fraction method:

Number of games won is 40% of 25.

Because 40% = 0.40

Then 0.40 x 25 = 10

The number of games won is 10.

CORE OBJECTIVES

b) to calculate what percentage one number is of another number;

NOTES TO TEACHERS

3. Unitary (one percent) method: (can be used to clarify student's thinking).

100% of 25 = 25

1% of 25 = $\frac{25}{100}$ or 0.25

Hence, 40% of 25 = 40 x $\frac{25}{100}$ or 40 x 0.25

= 10

The team won 10 games.

More difficult problems could be used depending on the class. For example, consider the following problem on efficiency.

Example:

A turbine in a hydro-electric plant converts power with an efficiency rate of 60%. If the power of the water that drives the turbine is rated at 850 000 kw (kilowatts), how much electrical power is generated by this turbine? [This problem could be expanded to a project in which students are required to contact Manitoba Hydro in order to obtain current data on northern Manitoba hydro development.]

This is a more difficult problem because of the increased reading level and the size of the numbers.

Students need considerable practice in finding what percent one number is of another before word problems are attempted. Problems and applications are readily found in consumer mathematics textbooks and the local environment.

Example:

A class contains 20 students. At the height of a flu epidemic 9 students were absent. What percentage of the students were absent?

1. Proportion method:

Let "n" = percent of students absent.

Then $\frac{9}{20} = \frac{n}{100}$

n = 45

45% of the students were absent.

CORE OBJECTIVES

NOTES TO TEACHERS

2. Fraction method:

Fraction of students absent is $\frac{9}{20}$

Percent of students absent is $\frac{9}{20} \times 100\% = 45\%$.

45% of the students were absent.

3. Unitary method:

20 students represent 100%

1 student absent is $\frac{1}{20} \times 100\%$

9 students absent is $9 \times \frac{1}{20} \times 100\% = 45\%$

45% of the students were absent.

* Suggested materials: newspaper advertisements, catalogues, sale flyers, MAB blocks, hundred cube tray, hundred square grid.

c) to calculate provincial sales tax on a specified purchase.

Students need practice in computing the retail sales tax for Manitoba. This practice should involve calculations as well as interpreting and using the Manitoba Retail Sales Tax Table. A portion of this table is found below.

AMOUNT OF SALE	5% TAX PAYABLE	AMOUNT OF SALE	5% TAX PAYABLE	AMOUNT OF SALE	5% TAX PAYABLE	AMOUNT OF SALE	5% TAX PAYABLE
$ .26 to .29	$ .01	$10.10 to 10.29	$ .51	$20.10 to 20.29	$1.01	$30.10 to 30.29	$1.51
.30 to .49	.02	10.30 to 10.49	.52	20.30 to 20.49	1.02	30.30 to 30.49	1.52
.50 to .69	.03	10.50 to 10.69	.53	20.50 to 20.69	1.03	30.50 to 30.69	1.53
.70 to .89	.04	10.70 to 10.89	.54	20.70 to 20.89	1.04	30.70 to 30.89	1.54
.90 to 1.09	.05	10.90 to 11.09	.55	20.90 to 21.09	1.05	30.90 to 31.09	1.55
1.10 to 1.29	.06	11.10 to 11.29	.56	21.10 to 21.29	1.06	31.10 to 31.29	1.56
1.30 to 1.49	.07	11.30 to 11.49	.57	21.30 to 21.49	1.07	31.30 to 31.49	1.57
1.50 to 1.69	.08	11.50 to 11.69	.58	21.50 to 21.69	1.08	31.50 to 31.69	1.58
1.70 to 1.89	.09	11.70 to 11.89	.59	21.70 to 21.89	1.09	31.70 to 31.89	1.59
1.90 to 2.09	.10	11.90 to 12.09	.60	21.90 to 22.09	1.10	31.90 to 32.09	1.60
2.10 to 2.29	.11	12.10 to 12.29	.61	22.10 to 22.29	1.11	32.10 to 32.29	1.61
2.30 to 2.49	.12	12.30 to 12.49	.62	22.30 to 22.49	1.12	32.30 to 32.49	1.62
2.50 to 2.69	.13	12.50 to 12.69	.63	22.50 to 22.69	1.13	32.50 to 32.69	1.63
2.70 to 2.89	.14	12.70 to 12.89	.64	22.70 to 22.89	1.14	32.70 to 32.89	1.64
2.90 to 3.09	.15	12.90 to 13.09	.65	22.90 to 23.09	1.15	32.90 to 33.09	1.65

Students could also calculate the value of some hidden taxes, such as federal sales tax, import tax, excise tax and customs duties.

CORE OBJECTIVES	NOTES TO TEACHERS
VIII. GEOMETRY AND MEASUREMENT	Geometry should be introduced at various points throughout the mathematics program. The word "geometry" means earth measure and in ancient times geometry was used in surveying and laying out land areas. Geometry was also used in the plans for the construction of the Pyramids and in irrigation projects along the Nile River.
1. Lines and Line Segments The student should be able:	Students should learn the geometric concepts which they will need to function effectively in a three dimensional world. The teaching approach should be inductive and experimental and should rely heavily on perception and intuition. It is important that the formal nature of the concepts not be imposed on the student. Grades 7-9 teachers should use geoboards, tangrams, paper folding, the Mira, polyhedra models, motion geometry as well as line designs and string art in their classes. Try to make geometry exciting for the students.
a) to identify examples of line segments from the environment;	Students should realize there are no "real-world" examples of the mathematical term "line" because examples found in the environment represent line segments. Examples may include markings on a football or baseball field, window frames, hydro lines viewed from the air, etc.
b) to estimate the length of a line segment in millimetres and in centimetres;	Estimation is based on previous experience. If students are unable to successfully estimate the lengths of line segments it will be necessary to provide workshop experiences.
c) to use a metric tape or ruler to measure line segments to the nearest millimetre, centimetre or metre;	Linear measurement is a necessary skill for all students. Also, students should develop the skill to estimate measurements. This skill is perfected by continuously making estimations and then comparing the results to the actual measurements. Linear measurements should be made in the classroom, in the school, in the community, both verbally and on paper. Students will have a good idea of how long a millimetre, centimetre and metre are. However, most students do not have the concept of a kilometre. Perhaps classes could measure off 20 metres. If this distance is walked 50 times, the total distance traversed is one kilometre. A kilometre should then be translated into a specific distance between two points in the community.
d) to classify lines as being horizontal, vertical or oblique;	The concept of vertical and horizontal is not easy for many students. Care must be taken to review this idea after it is understood. Students could learn hand signals for vertical, horizontal, oblique, parallel and perpendicular line segments and then play "Simon Says". The teacher could participate both as Simon or as a contestant. This activity is a take-off on the old game, "Do This, Do That".
e) to identify a pair of lines as being parallel or perpendicular;	Window and door frames can be used to indicate both parallel and perpendicular line segments.

CORE OBJECTIVES

f) to identify the lines of symmetry for segments;

NOTES TO TEACHERS

NOTE: Teachers may choose to deal with symmetry as a separate topic rather than integrate it throughout the study of geometry as outlined in this guide.

Symmetry ideas span a wide spectrum of use and understanding. Artists use symmetry in form and colour to create moods and impressions, architects use symmetry in design, and scientists look for patterns of symmetry in everything from plant structure to crystallography. Line or bilateral symmetry will be considered in this first section.

Students could determine the line or axis of symmetry for a given line segment by paper folding or by using a Mira or mirror. Ultimately, students should realize that the perpendicular bisector of a line segment is its line of symmetry. Time may need to be spent exploring the concepts of bisecting and perpendicular bisector.

Students could determine which letters of the alphabet, written in block form, have one or more lines of symmetry. This could be carried one step further to include words such as:

H
A
T

or

M
A
T
H

or BOX

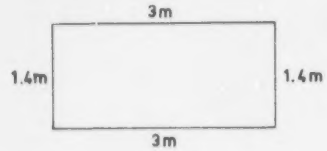
Students could be encouraged to find other such words. Students could consider palindromes (words which read the same backward and forward) at this time. The palindrome 1881 has two lines of symmetry. Find other symmetric numbers. Also, animal heads and human faces are nearly symmetric.

Example:

Give the next symbol in the following sequence:

Solution:

If a line of symmetry is drawn vertically through the middle of each figure it is evident that the symbols represent the numerals 1, 2, 3 and 4. Hence the next symbol must represent 5 and would be .

CORE OBJECTIVES	NOTES TO TEACHERS
2. Perimeter The student should be able to determine the perimeter of any polygon through measurement or through computation when given the lengths of the sides.	NOTE: Students have a great deal of difficulty distinguishing between the concepts of perimeter and area. Teachers are reminded to deal with these concepts separately. The word "perimeter" is derived from Greek and means "around measure". Formulae may be developed but the major emphasis should be on finding the perimeter the easiest way possible. Stress measurement and not formulae. Practical problems relating to perimeter include fencing fields or lots and weather stripping windows or doors. Students should be required to solve problems pertaining to perimeter. Students should draw diagrams whenever possible. Example: Find the perimeter of a rectangle having a length of 3 metres and a width of 140 centimetres. Solution: Draw a diagram. All dimensions must be converted to the same unit. 140 cm = 1.4 m P = 3 + 1.4 + 3 + 1.4 = 8.8 The perimeter is 8.8 metres. 
3. Area The student should be able to determine the area of any square, rectangle or parallelogram.	Teachers are reminded to review briefly non-standard units of area. See the K-6 Mathematics Guide pages 124, 167-168, 223, 291 for suggestions about area. Students should observe that the measurement of the interior region of a figure is called its area. Although any shape could be used to measure area, for convenience a square is used. The units used to measure area must be the square units (i.e., mm^2, cm^2, m^2 or km^2). Area is determined by counting the number of such squares. Ultimately students are to realize that area is the number of rows times the number of square units per row.

CORE OBJECTIVES

NOTES TO TEACHERS

Example:

What is the area of the following figure in square centimetres if each square is 1 cm by 1 cm?

Solution:

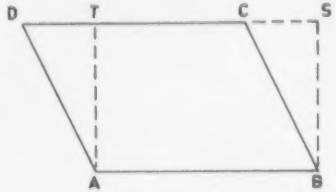
Area = rows x squares per row

$A = 3 \times 4\ cm^2$

$A = 12\ cm^2$

NOTE: Students should express the area in square centimetres abbreviated as cm^2. The concept of cm x cm which is written cm^2 causes confusion for many students.

The concept of area for a parallelogram is new for grade 7 students. The easiest way to show concretely how to find the area of a parallelogram is to convert the parallelogram into a rectangle as shown in the following diagram.



Remove triangle ATD from parallelogram ABCD. Place triangle ATD in the position of triangle BSC in the diagram creating rectangle ABST. Thus, students should realize that the area of the parallelogram is the same as that of the corresponding rectangle.

In order to maintain that the basic unit for measuring area is a square it should be stressed that the base measure and the height measure must be mutually perpendicular.

In order to check if students understand the concept of area, non-regular figures such as leaves, hands or irregular closed curves could be traced on squared paper. The number of unit squares required to cover the figure(s) should be estimated and then counted.

The formulae for the area of a square or a rectangle should be introduced only after students have developed the concept of area.

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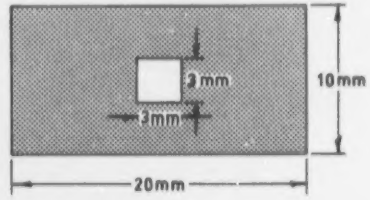
CORE OBJECTIVES

NOTES TO TEACHERS

Teachers could discuss the hectare as a unit of land measure in the SI system. The following facts are useful in the development of the concept of a hectare: 1 centare = 1 square metre; 1 are = 100 square metres; 1 hectare = 10,000 square metres.

Examples:

1. Find the area of the shaded region on the sheet metal plate drawn below.



Solution:

Area of rectangle = 10 x 20 = 200 mm^2
Area of square = 3 x 3 = 9 mm^2
Area of shaded section = 200 - 9 = 191 mm^2

2. A rectangular field measures 825 metres by 932 metres. What is the area of the field in hectares?

Solution:

Students should estimate an answer, calculate the answer and then compare the two answers to this problem.

4. Angles

The student should be able:

There are two ways to think of angles:

1. as the union of two rays having a common end point;
2. as the active rotation of a single ray about its end point. In this definition it is necessary to refer to the initial and the terminal positions of the arm of the angle.

CORE OBJECTIVES

a) to identify the vertex and sides of an angle;

b) to name an angle using a proper notation;

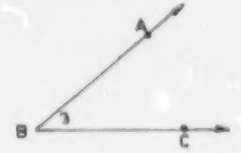
c) to measure the size of an angle whose measure is less than 180^0, using a protractor;

d) to draw any angle, whose measure is less than 180^0, using a protractor;

NOTES TO TEACHERS

Students usually find it easier to acquire the angle concept when it is presented using the second definition. This representation makes the idea of angles measuring over 180^0 plausible.

Angles are important in construction, urban planning, flight planning, curling, baseball, billiards, Crokinole, art, etc.



This angle could be denoted by ∠ABC, ∠3, ∠CBA or ∠B. However, the notation ∠B, is not favoured because it is unacceptable when more than one angle has B as its vertex.

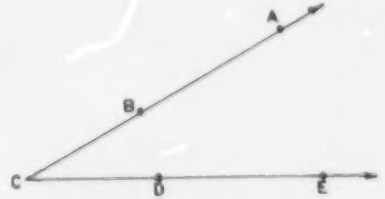
The process of measuring is important. Practice with measuring instruments is essential. Students must be encouraged to estimate and then to record the actual measurement and also must be reminded to include the symbol for the unit of measurement.

The word and symbol for "degree" is the same in temperature and angle measure. This causes problems for many students.

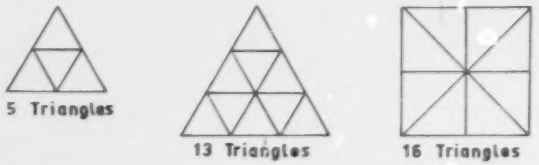
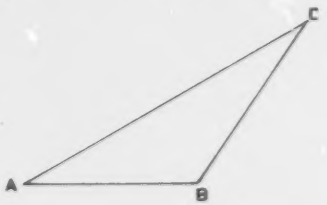
Because two scales are marked on the 180^0 protractor, one must be extremely careful to use the correct scale. Students need practice in measuring angles in both clockwise and counter clockwise directions. Students must realize that the size of an angle is independent of the length of its arms.

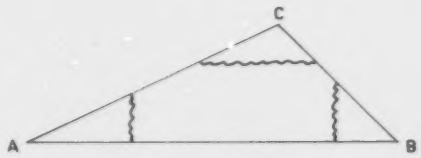
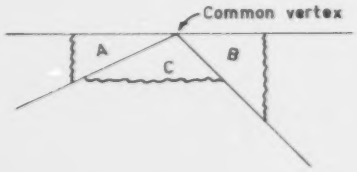
Example:

∠ACE has the same measure, so is the same size, as ∠BCD in the accompanying sketch.



Variations can be achieved by measuring the angles between the hands of a clock or by determining the measure of the angles in various polygons, sundials, alidades, compass bearings, etc.

CORE OBJECTIVES	NOTES TO TEACHERS
e) to identify angles according to their size, as being acute, right, obtuse or straight;	The importance and use of right angles should be stressed.
f) to identify the line of symmetry for any angle.	Paper folding can be used to determine the line of symmetry for an angle. Students should discover that the angle bisector is the line of symmetry for an angle. The term angle bisector may be new to some students.
5. Triangles	
The student should be able:	In elementary school, students used models of triangles so their concept of a triangle is likely to include the inside as well as the edges. Thus, grades 7-9 teachers should concentrate on identifying the types and properties of triangles rather than on formally defining triangles as only the boundary of the figure. What does the word "triangle" mean? Three angles. Triangles are of great importance in geometry because many useful plane figures can be divided into triangles. Each diagonal placed in a quadrilateral increases the number of triangles formed therein. Also, triangles exist all around us. Encourage students to give examples of triangles found in the environment. Students enjoy making posters similar to the following and then counting the number of triangles. 
a) to use the proper notation to name a triangle;	The triangle shown below can be named $\triangle ABC$, $\triangle ACB$, $\triangle BAC$, $\triangle BCA$, $\triangle CAB$, $\triangle CBA$. 

CORE OBJECTIVES	NOTES TO TEACHERS
b) to determine the perimeter of a triangle;	
c) to measure each of the angles of a triangle using a protractor;	Students should be given opportunity to discover and understand the relationship, "the sum of the measures of the angles in a triangle is 180°". If students realize that the measure of a straight angle is 180°, a verification of the preceding relationship can be determined by constructing a large cut-out triangle, tearing off the three corners and placing them so that they have a common vertex. The three corners of the triangle will form a straight angle as shown below.  
d) to classify triangles by sides and angles;	sides: scalene, isosceles, equilateral angles: acute, right, obtuse Students should be able to combine the classifications of a triangle by sides and angles to get, for example, an obtuse isosceles triangle. Students should discover, by measurement, that if a triangle has three equal sides, it also has three equal angles. If a triangle has two equal sides, it has two equal angles and the equal angles are opposite the equal sides.
e) to identify the lines of symmetry for triangles;	Students should demonstrate that triangles with one line of symmetry are isosceles, triangles with three lines of symmetry are equilateral and triangles with no lines of symmetry are scalene. Where lines of symmetry exist they are angle and side bisectors.
f) to identify rotational or point symmetry;	If a plane figure coincides with itself under a partial revolution, the figure is said to have rotational symmetry. It is important to note that the figure must remain flat on the page (in the same plane); it cannot be flipped over. The point or centre about which the figure rotates is called the center of rotation. The order of rotational symmetry refers to the number of times that the figure can be superimposed or mapped upon itself in one complete revolution. To begin a study of rotational symmetry, consider the three types of triangles (scalene, isosceles, equilateral). Other examples should be selected from the

CORE OBJECTIVES	NOTES TO TEACHERS

letters of the alphabet, numbers, polygons, flowers, children's pinwheels, fans, windmills, bottle caps, etc. Students should manipulate objects to test for rotational symmetry.

Examples:

In each of the following, the order of rotational symmetry is identified.

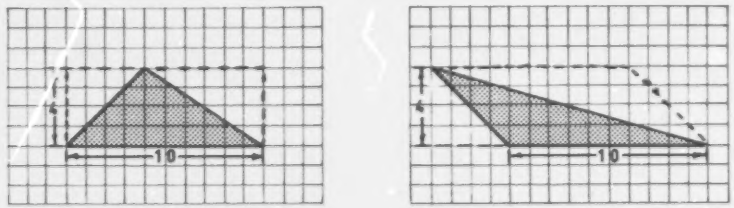
Tesselations and floor tiling problems could be considered here. Students could also investigate the symmetries of regular polygons and of three-dimensional figures.

g) to determine the area of a triangle.

Students could be introduced to this topic using their knowledge of area. Rectangles could be drawn on graph paper. A triangle, on the same base and with the same height, would then be drawn within the rectangle. The area of the rectangle could be determined by counting or by formula. If cut-out's are made, it is easy to show that the unshaded triangles within the rectangle cover the same area as the shaded triangle. After many such examples students can generalize that the area of such a triangle is one-half the area of the rectangle. [In order to find the area of an obtuse triangle it would be best to use a parallelogram rather than a rectangle].

Example:

Determine the area of the shaded triangle in the accompanying diagrams.



CORE OBJECTIVES

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Solution:

Area of rectangle or parallelogram
= 10 x 4
= 40 square units.

The area of the shaded triangle can be shown, as described above, to be one-half the area of the rectangle or parallelogram.

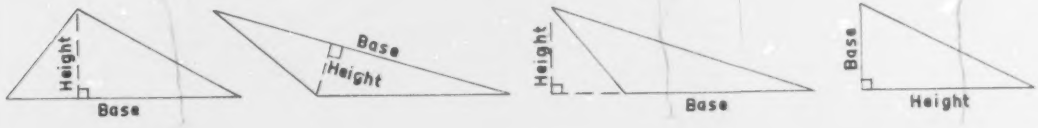
Area of triangle = $\frac{1}{2}$ x (Area of rectangle or parallelogram)

= $\frac{1}{2}$ x 40

= 20 square units.

Before students are ready to use the formula, $A = \frac{1}{2}bh$, considerable time must be spent identifying and measuring the base and the height of various triangles in different positions. It is important to note that any side of a triangle can be considered the base.

Examples:



Students are not required to memorize formulae.

Teachers are reminded to use estimation and measurement activities together throughout the geometry program.

Students should determine the area of a given triangle considering each side of the triangle as the base. A concrete model would be advantageous.

* Suggested materials: graph paper, geoboard, geopaper, centimetre cubes, mirrors, Mira, tracing paper.

CORE OBJECTIVES

6. Motion (Transformational) Geometry

The student should be able to identify and to illustrate slides (translations), flips (reflections), turns (rotations) and combinations of these rigid motions.

NOTES TO TEACHERS

NOTE: All students are to be exposed to the motion or transformational geometry concepts outlined in this guide. This is not an optional topic.

Some reasons for motion geometry include the following:

1. Everyday experience. Students should become consciously aware and able to classify the motions around them.
2. Manipulative activities. A laboratory approach to motion geometry includes many visual and tactile experiences.
3. Congruency and similarity which are traditionally studied in the context of triangles can be learned more intuitively through transformations and symmetry.
4. Symmetry types are easily expressed as transformations.
5. Most transformational geometry appears "new" to students and it is important to expose students to new ideas and thereby broaden their outlook.

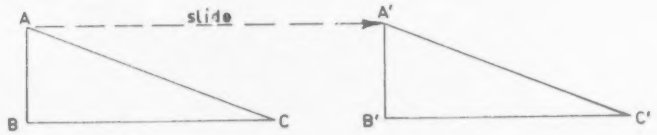
Students have been exposed to motion geometry concepts in elementary school. These transformations are called rigid motions because there is no change in the shape or size of the original figure.

Students are expected to identify these motions or transformations from their environment. Every attempt should be made to relate the three motions with life motions and various symmetries. Bilateral or line symmetry relates to reflections. Students should use a mirror or a Mira (trade name for a semi-transparent plastic device with mirror qualities) when working on flips or reflections. Point symmetry relates to rotations.

Suggestions for introducing the transformations include:

1. slide - use concrete objects such as a game of chess.

   Use slide arrows to indicate the direction and distance of the slide.

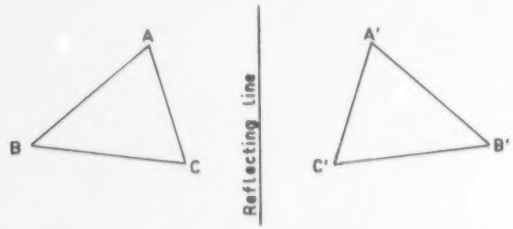


CORE OBJECTIVES

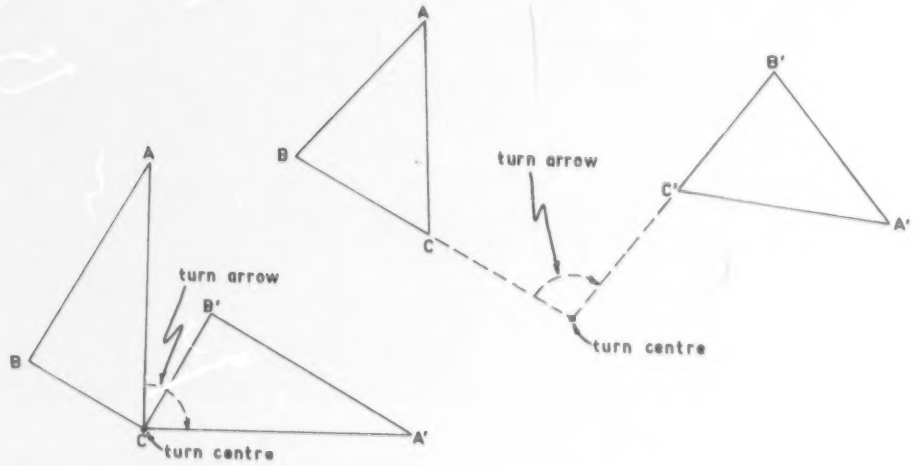
NOTES TO TEACHERS

2. flip - use a semi-transparent mirror (Mira), tracing paper or paper folding.

   Use a reflecting line to indicate a "flip". Note that a point of the object, and the image of that point are equidistant from the reflecting line.



3. turn - use a turn centre and a turn arrow to indicate the direction of the turn. Introduce the turn by having the turn centre at a point on the figure, at a point inside, and at a point outside the figure.



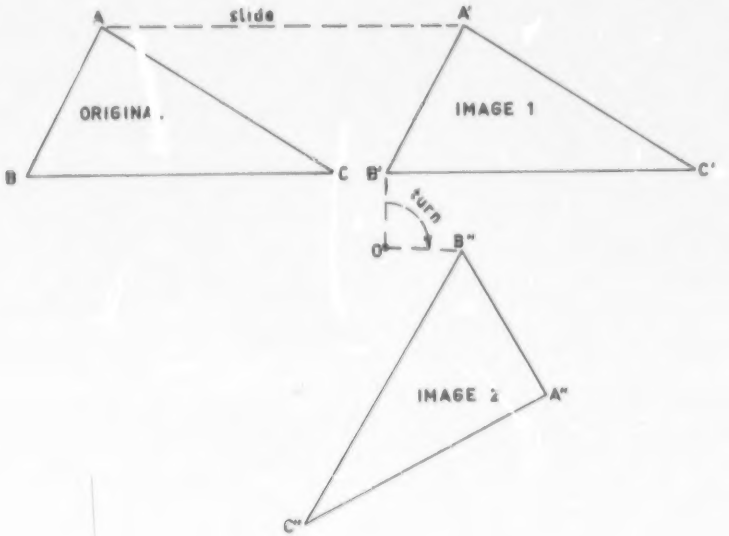
NOTE: Initially use only quarter, half, and three-quarter turns. An overhead projector is particularly effective for illustrating transformations.

CORE OBJECTIVES

NOTES TO TEACHERS

4. combination - use only two motions.

Example:



IX. GRAPHING

1. Data Graphing

The graphing portion of the course could be done at any time and should be integrated with the work in other subject areas.

This is basically review work.

Graphs are one of the most useful forms for interpreting data and for displaying information. Today, pictorial representations of data are evident in all media forms. Therefore, not only is it important that students be able to construct graphs but also they should be able to read and to interpret graphs. Graphical representations are an important means of communication and provide an additional method for developing student interest and motivation. This section of the grade 7 program can be made more meaningful by using examples from social studies, science, physical education, home economics, industrial arts and current newspapers or magazines.

Students should have experience in gathering data both by counting (called discrete data) and by measuring (continuous data). Suggestions include school elections, times or distances for track and field events, school fund raising projects, and scores from both intramural and intermural sports contests.

CORE OBJECTIVES

NOTES TO TEACHERS

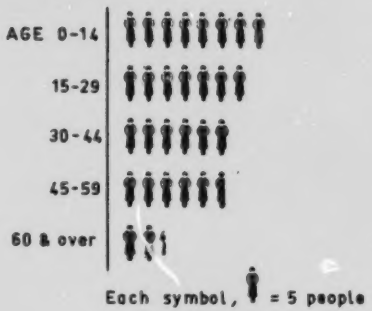
NOTE: Before attempting to construct graphs students should learn to interpret them.

The student should be able:

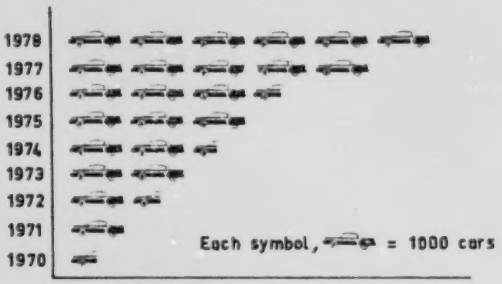
a) to interpret and to construct a pictograph;

A pictograph may also be called a pictogram or a picture graph. Drawings may be very simple or quite complex as shown in the following examples.

PICTOGRAPH SHOWING
AGES OF RESIDENTS
IN SMALL TOWN



PICTOGRAPH SHOWING
NEW CARS SOLD IN MANITOBA
FROM 1970 - 1978



b) to interpret and to construct a bar graph;

Any records which show comparisons for more than one time period usually provide information suitable for making a bar graph. Students should consider bar graphs employing both vertical and horizontal bars.

Constructing a bar graph requires making the following decisions:

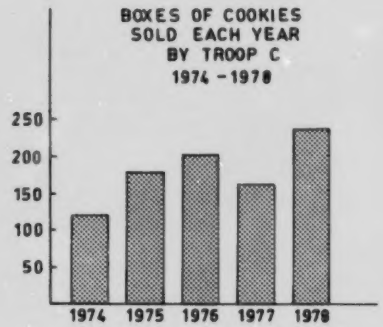
1. What width will the bars be?
2. Will the bars be vertical or horizontal?
3. How widely spaced should the bars be?
4. What scale should be used on each axis?
5. How will the axes be labelled?
6. What title should be used?

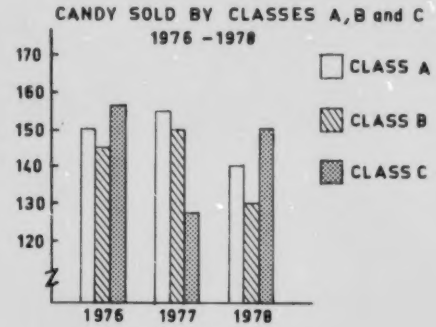
CORE OBJECTIVES

NOTES TO TEACHERS

c) to interpret and to construct a line graph.

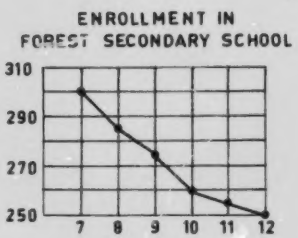
Examples:

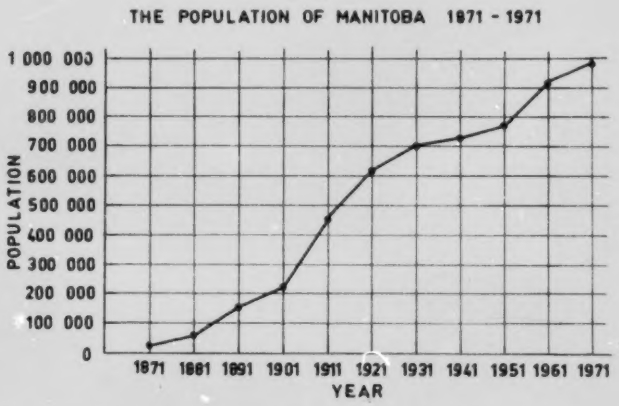




Line graphs are often used to show change over a period of time. The vertical scale usually indicates the change while the horizontal scale shows the time. One suggestion is to have students keep data on their own height during the grade 7 year. Perhaps measurements could be made each month. The line graph could be completed in June.

Examples:





Teachers could use data from Environment Canada pertaining to local weather conditions as one source for graphing exercises. Another source is an atlas. The data from an atlas provides greater variation in the range of temperature and precipitation data.

CORE OBJECTIVES

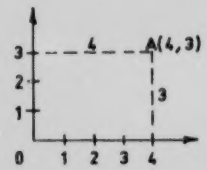
2. Coordinate Graphing

The student should be able to locate points and to identify coordinate pairs in the first quadrant.

NOTES TO TEACHERS

Establish the need for locating a point on a plane. Students should be encouraged to give real world uses for coordinate graphing. These could include fixing points on Forest Service maps, map reading, locating objects on a radar screen, determining distance between points on a map, enlarging and reducing drawings, planning cheering section designs involving spectators at sports events, drawing picture graphs on a coordinate system or playing games such as Battleships.

Students need to be familiar with the terminology of coordinate graphing - ordered pair, coordinate, axis, origin. In order to create interest, teachers could encourage students to research the history of coordinate graphing. The Cartesian coordinate system is named for René Descartes. Descartes' invention was clever, yet very simple. The idea was this: draw two lines perpendicular to each other. The position of any point can be given by telling its distance from these lines or axes.



The coordinates for point A are (4,3). The first coordinate gives the distance from the vertical axis while the second coordinate gives the distance from the horizontal axis.

Motivational ideas include:

1. picture plotting. Ideas may be obtained from books on this subject or students could prepare examples for each other. Useful suggestions include using block letters which spell student names, drawing diagrams of apparatus found in a science classroom (test tube, flask, balance, burner, etc.), and preparing a design for use by spectators at the school's next sports rally.

Example:

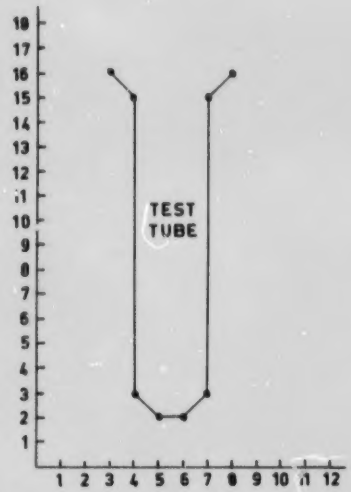
Plot the following ordered pairs on a graph and identify the diagram:

(3,16), (4,15), (4,3), (5,2), (6,2), (7,3), (7,15), (8,16).

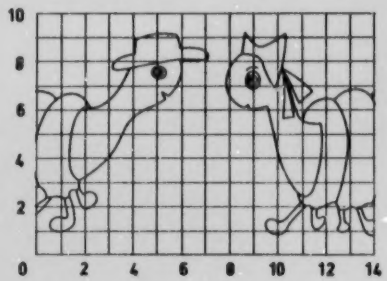
CORE OBJECTIVES

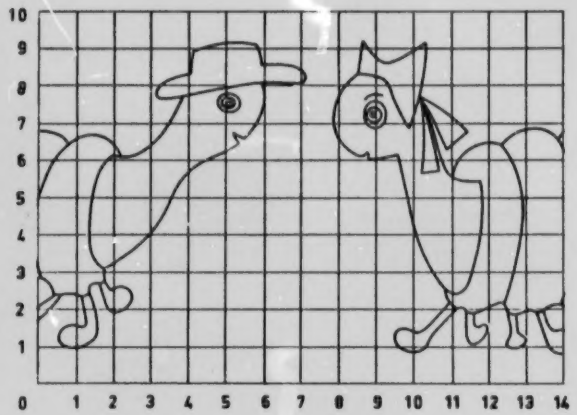
NOTES TO TEACHERS

Solution:



2. enlargement or reduction of a picture. Provide a picture on a grid and have the students enlarge or reduce the picture by transferring to a larger or smaller grid. Pictures already on a grid are found in needlework patterns, some paint by number kits, dress patterns, blue prints and patterns for carpenters, etc.





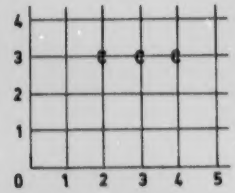
CORE OBJECTIVES

NOTES TO TEACHERS

3. battleships suitably modified. The commercial games which employ a coordinate system often represent a square region by an ordered pair. It is suggested that this not be allowed in a mathematics class. The ordered pair should represent a point or one corner of the squared region.

Example:

The ordered pairs (2,3), (3,3) and (4,3) represent three cruisers on the accompanying diagram.



Grade 8

GRADE 8 MATHEMATICS GUIDE

The following guide identifies the core objectives for regular grade 8 students. Teachers are reminded that these objectives constitute a minimum program and that enrichment topics should be included, whenever possible.

Enrichment topics could include: the history of optical illusions, perspective drawings, calculators and computers, puzzle and problem solving, outdoor mathematics activities, preparation for mathematics contests, mathematics applications found in the local community, historical mathematics development, measuring instruments, alternate computation methods, logic activities, topology, colour theorems, geometric models, paper folding activities (origami), mathematical activities and experiments, Königsberg bridges and other network problems, paradoxes, mathematics posters, probability, readings in mathematics, mathematics projects, tangrams, or permutations and combinations.

INDEX TO TOPICS

I. WHOLE NUMBER SYSTEM

1. Fundamental Operations
2. Divisibility
3. Review of Number Theory

II. INTEGERS

1. Meaning of Integers
2. Operations
3. Problem Solving

III. ALGEBRA

1. Use of Variable
2. Algebraic Operations
3. Solving Equations
4. Problem Solving

IV. RATIONAL NUMBERS

1. Identification
2. Conversions
3. Operations with Rationals

V. RATIO AND PERCENT

1. Ratio and Proportion
2. Percent
3. Estimating Percents
4. Percent Problems
5. Applications

VI. GEOMETRY

1. Measurement
2. Quadrilaterals
3. The Circle
4. Motion (Transformational) Geometry

VII. COORDINATE GRAPHING

1. Coordinate Plane
2. Locating Points

CORE OBJECTIVES

NOTES TO TEACHERS

Teachers of grade 8 mathematics should familiarize themselves with the core objectives outlined in the grade 7 mathematics guide. In addition, there should be discussion between the receiving teacher and all feeder teachers regarding the material mastered by incoming students.

NOTE: See the section entitled "Unifying Themes" on pages 18-19. The skills identified under "Unifying Themes" are of such importance that teachers must emphasize their use at every opportunity.

Teachers are requested to examine the core objectives outlined in the grade 8 guide and to determine the best order for teaching the outlined topics. Teachers are advised to integrate geometry and graphing exercises throughout the course rather than to deal with them as separate topics.

I. WHOLE NUMBER SYSTEM

It is always a good idea to begin a new year's work with new content. Although many teachers prefer to review arithmetic operations during the first few weeks of a new school term, for many grades 7-9 students there is often nothing new or exciting about computation. One way to catch the interest of students is to introduce them to new, but easy, ideas from different areas of mathematics so that some of the needed computation, review and practice can be hidden within investigations of higher-level concepts. Students of all ages and abilities enjoy pondering over questions that do not seem too difficult, especially if the lead-up questions have been relatively easy. Topics could include sequences and series, statistics, consumer mathematics, measurement geometry or alternate computation methods.

1. Fundamental Operations

Questions involving the four fundamental operations of addition, subtraction, multiplication and division of whole numbers should be reviewed, if required.

The student should be able to demonstrate proficiency in performing operations with natural and whole numbers:

a) by determining the next number in a given number sequence;

Examples:

Determine the next whole number in each of the following sequences:

1. 8, 12, 17, 24, 28, 33, ____. (40)
2. 3, 12, 6, 24, 12, 48, ____. (24)
3. 8, 7, 10, 5, 4, 7, ____. (2)
   [The question stipulates "next whole number" so the answer $3\frac{1}{2}$ is eliminated.]
4. 90, 45, 50, 25, 30, 15, ____. (20)
5. 5, 3, 9, 7, 21, 19, ____. (57)

CORE OBJECTIVES	NOTES TO TEACHERS
	Sequences with more than one correct answer should also be used. Be sure students understand how to arrive at both possible solutions. 6. 24, 12, 36, 24, 48, 36, ____. (60 or 36) 7. 15, 11, 7, 14, 10, 6, ____ (12 or 13) Teachers can obtain examples of sequences from textbooks, make them up, or encourage students to create such sequences. Students could challenge their classmates to determine the next number(s) in their sequence(s).
b) by calculating the arithmetic mean and median for a set of data.	Calculating the mean involves the operations of addition and division with rounding-off in some cases. Students should be encouraged to gather their own data for computation purposes. Calculators may be used to ease calculation and to motivate students, particularly if large samples are being used. NOTE: The arithmetic mean is only one form of average. The three common averages are mean, median and mode. Encourage students to find examples in newspapers and magazines showing how the term "average" is employed. Where possible, identify the type(s) of average. Do any of the articles "falsify" with their misuse of statistics? Teachers might also encourage students to group data into frequency distribution tables and use the grouped data to calculate the approximate arithmetic mean. So-called "street mathematics" should be considered. For example, if a ball player is batting 0.233 going into a game and he gets three hits in four times at bat (which means he has batted 0.750 for the day), a student might assume that the day's performance will make a big improvement in his batting average. But it may turn out that the three-for-four day only raises the 0.233 to 0.242. Disappointing, but a good personal lesson in fractions, ratios and percents. * Suggested materials: newspapers, atlas, Statistics Canada data, sports data.
2. Divisibility The student should be able to test any number for divisibility by 2, 3, 4, ..., 10.	This may be a new topic for grade 8 students. Students are to be encouraged to use the divisibility rules mentally. The suggested order for learning the divisibility rules is: 5, 10; 2, 4, 8; 3, 9, 6; 7. The ability to tell at a glance whether a number is divisible by any of the listed numbers is a real asset in determining the factors of a number. A whole number is divisible by: 5 if it ends in 0 or 5; 10 if it ends in 0;

CORE OBJECTIVES

NOTES TO TEACHERS

2 if the units digit is divisible by 2 (i.e., it is even);
4 if the number formed by considering the last two digits is divisible by 4;
8 if the number formed by considering the last three digits is divisible by 8;

This could be expanded to 16, 32, 64,...

3 if the sum of its digits is divisible by 3;
9 if the sum of its digits is divisible by 9;
6 if it is even and divisible by 3; [Why? 6 is a composite of 2 and 3. In order for it to be a factor of another number, that number must be divisible by 2 and by 3.]

Divisibility tests for other composite numbers could be developed. Try 12 (divisible by 3 and 4 rather than 2 and 6), 15 (divisible by 3 and 5), 20 (divisible by 4 and 5), 21, etc.

7 if the difference between twice its units digit and the number formed from the remaining digits is divisible by 7. Divisibility by 7 should not be stressed.

Example 1:

Is 632 divisible by 4?

Solution:

Consider last two digits, "32". Since 32 is divisible by 4 then so is 632.

Example 2:

Use the appropriate divisibility rule to show whether 1806 is divisible by 7.

Solution:

Find the difference between twice the units digit and the remaining digits.

$180 - 2(6) = 180 - 12$

$= 168$

But, $168 \div 7 = 24$ so the original number, 1806, is divisible by 7.

CORE OBJECTIVES	NOTES TO TEACHERS
3. Review of Number Theory The student should be able to demonstrate proficiency in the fundamentals:	Students are expected to achieve computational proficiency for whole numbers in each of the following areas. This includes understanding as well as the ability to express both oral and written responses in each topic area. For example, when dealing with factors, the ultimate stress is to be placed on the ability to determine the factors of a given number. The content of this section should be reviewed and maintained. It is essential that a variety of drill techniques be used throughout the year to achieve and to maintain proficiency.
i. place value,	See grade 7 guide page 36 .
ii. expanded notation,	
iii. rounding-off and estimation,	The importance of rounding-off and estimation as a mental exercise cannot be over-stressed. Teachers must constantly encourage students to estimate answers before solving problems. In order to ensure that students are estimating answers and then solving questions rather than first solving and then rounding-off the answers, it is suggested that, on occasion, students turn in all estimated answers and then proceed to solve the problem(s). Stress must also be placed on comparing estimated and computed answers to various questions.
iv. prime and composite numbers,	When considering the whole numbers, "0" and "1" are generally considered to be neither prime nor composite. Students are expected to be able to determine the prime factors for a composite number through multiplication or division.
v. LCM and GCF for two or more whole numbers.	See grade 7 guide pages 46, 47.
II. INTEGERS	Integers is a new topic for grade 8 students. Teachers should discuss the need for positive and negative numbers. Examples might include elevation, profit and loss, temperature, barometric pressure, football games, assets and liabilities, time before and after rocket lift-off, north and south latitude, forward and backward directions, etc. The study of integers or directed numbers is an integral part of the study of algebra. However, operations with integers is a difficult topic to develop successfully. The fact that students fail to obtain adequate mastery of the fundamental operations with integers accounts for frequent difficulties which they experience in the study of algebra.

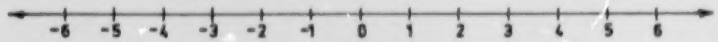
CORE OBJECTIVES

NOTES TO TEACHERS

1. Meaning of Integers

The student should be able to identify the set of integers.

$I = \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$



When considering the number line it is necessary to give zero a new significance. In addition to its use as a number and as a place holder, zero can now be regarded as the starting point on a number line from which one can measure in either direction. Also, while some texts use the elevated signs $^{+}8$ or $^{-}8$ students should be encouraged to use 8 and -8 as soon as possible.

2. Operations

NOTE: There is a difference of opinion over the extent to which teachers should attempt to rationalize and to explain the operations with integers. Since this topic represents an extension of the number system, the new cannot be explained entirely in terms of the old. Do not obscure the concept with the illustration.

Aim for student mastery of the operations with integers.

The student should be able:

a) to identify the additive inverse (opposite) of any integer;

Students should understand that $3 + (-3) = 0$ and that 3 and -3 are opposites or additive inverses. Students should realize that the additive inverse of any integer, other than 0, always has the opposite sign. Students are not required to memorize properties.

b) to add and to subtract integers;

Addition:

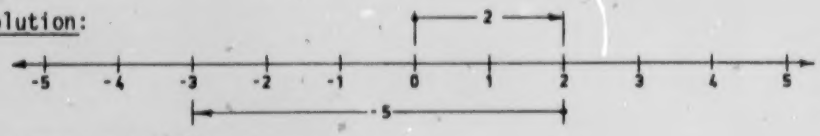
Carefully re-emphasize that addition is a process of "combining" or "putting together". Addition of integers can be taught using:

1. A Number Line

Example:

Use a number line to determine the value of $2 + (-5)$.

Solution:



Hence, $2 + (-5) = -3$.

CORE OBJECTIVES

NOTES TO TEACHERS

Teachers must emphasize that the first number in the question tells how far and what direction to move from the "0" or starting point. From there, the second number indicates the direction and the distance which remains to be moved. Where you stop is the answer. Initially students should draw diagrams. Then students should attempt the number line movements mentally. Finally, students are expected to become proficient in performing the addition of directed numbers without reference to the number line.

Easy as this may appear, addition of integers requires much practice in order to fix the procedures firmly in the students' minds.

2. Arithmetic Reasoning

Example:

Determine the value of $2 + (-5)$

Solution:

$$\begin{aligned} 2 + (-5) &= 2 + [(-2) + (-3)] \\ &= [2 + (-2)] + (-3) \\ &= 0 + (-3) \\ &= -3 \end{aligned}$$

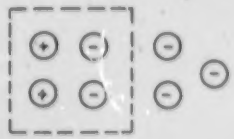
This method for addition of integers is included only to show why the procedures work. Students are not expected to become proficient in this method.

3. Concrete Approach

Example:

Use two colours of bingo or poker chips to find the value of $2 + (-5)$.

Solution:



Let one colour of chip represent + and the other colour -. Equal numbers of + and - chips combine or form zero. Whatever is left represents the answer.

Hence, $2 + (-5) = -3$.

CORE OBJECTIVES

NOTES TO TEACHERS

Subtraction:

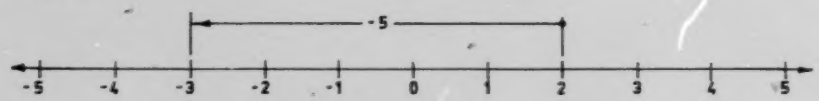
Students must be aware that addition and subtraction are inverse operations. Teachers might approach the problem as follows. Ask students to answer a number of subtraction questions, such as 50 - 33. Then, ask students how they could check to see if their answers are correct. Draw out the response that addition can be used to check subtraction questions. It is easy to show that if 50 - 33 = n, then n + 33 = 50. Subtracting 33 from 50 can then be defined, in a general way, as the process of finding what number must be added to 33 in order to yield 50.

The number line can be used to illustrate subtraction with integers.

Example

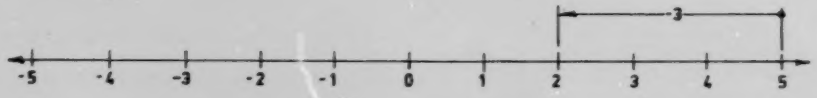
Use a number line to determine 2 - 5.

Solution 1:



Start at 2, count 5 units in a negative direction and read the answer -3 as shown.

Solution 2:



Students must realize that they need to determine what number added to 5 will give 2 so they must begin at 5 on the number line and go to 2. Therefore the answer is -3 as shown on the diagram. Or if 2 - 5 = n then n + 5 = 2 and n becomes -3.

Establish the algorithm that subtraction means adding the opposite. Numerous examples exhibiting various combinations of positive and negative numbers should be used.

CORE OBJECTIVES	NOTES TO TEACHERS

Examples:

Determine the value of each of the following:

1. $2 - 5 = 2 + (-5)$
   $= -3$
2. $2 - (-5) = 2 + (5)$
   $= 7$
3. $-2 - 5 = -2 + (-5)$
   $= -7$
4. $-2 - (-5) = -2 + (5)$
   $= 3$

c) to multiply and to divide integers.

Multiplication:

In the multiplication of integers the so-called "sign laws" are easily stated but their rationalization can be complicated.

Establish that:

1. a positive times a negative is a negative,
2. a negative times a negative is a positive.

Teaching strategies include:

1. Repeated Addition

   This strategy is effective if the multiplier is a positive integer.

   Example:

   $(-2) \times 3 = (-2) + (-2) + (-2)$
   $= -6$

2. Patterns

   Use the following patterns to predict the product:

   a) $3 \times 3 = 9$
   $3 \times 2 = 6$
   $3 \times 1 = 3$
   $3 \times 0 = 0$
   $3 \times (-1) = ?$
   $3 \times (-2) = ?$
   etc.

CORE OBJECTIVES

NOTES TO TEACHERS

b) (-3) x 3 = -9
(-3) x 2 = -6
(-3) x 1 = -3
(-3) x 0 = 0
(-3) x (-1) = ?
(-3) x (-2) = ?
etc.

NOTE: That these patterns are useful only for predicting the product.

3. Intuitive Story

Consider a town where good and bad people are moving in and out. Let a good person be + and a bad person be -. Moving in is + and moving out is -.

If we consider the first sign in a multiplication problem as representing the kind of person and the second sign as representing moving in or out then the sign of the product could represent whether the result was good or bad for the town.

Cases:

1. - x - = + since a bad person leaving town is a gain for the town.
2. + x + = + since a good person entering the town is a gain for the town.
3. + x - = - since a good person leaving town is a loss for the town.
4. - x + = - since a bad person entering town is a loss for the town.

Fourteen different strategies for multiplication of integers are found in an article by John C. Peterson in the May, 1972 issue of The Arithmetic Teacher.

Do not stress the idea of the intuitive story outlined above.

4. Deductive Approach

This strategy employs a number of field properties in its development. Not all students should be subjected to this approach.

Example:

Show that 7 x (-5) = -35.

CORE OBJECTIVES

NOTES TO TEACHERS

Solution:

$7 \times 0 = 0$

$7 \times [5 + (-5)] = 0$

$(7 \times 5) + [7 \times (-5)] = 0$

But, $35 + (-35) = 0$

Therefore, $7 \times (-5) = -35$.

Division

Introduce division as the inverse of multiplication. For example, it is said that $8 \div 2 = n$ if $n \times 2 = 8$. A quotient can thus be viewed as a missing factor in the same way as a difference is thought of as a missing addend. It is not difficult to develop the sign rules for division once division is established as the inverse of multiplication. Develop patterns to show that:

1. a positive divided by a negative is a negative,
2. a negative divided by a negative is a positive.

* Suggested materials: Thermometer, bingo chips, number line, Games - Integer Snap, Integer Metric Football, Postman (see Madison Project), Shake-up Game, Colour or Black Game.

3. Problem Solving

The student should be able to solve simple problems involving:

i. income and expenditures,
ii. bank balances,
iii. temperatures,
iv. weights.

See note on "Problem Solving" page 19.

Because integers can be used to describe quantities that are of the same magnitude but that are in opposite directions from a reference point, the system of integers is useful in many practical ways. Additional uses for integers are found at the beginning of this section on page 101.

Following the introduction of integers, teachers may wish to incorporate coordinate graphing exercises which have been expanded to include all four quadrants.

III. ALGEBRA

1. Use of Variable

The student should be able:

All students may eventually study the same algebra, but not in the same depth, nor at the same time, nor in the same sequence. It is not intended that the algebraic work in grade 8 will lead to a high degree of technical skill in algebraic operations. Rather, it is intended to serve the dual purpose of extending the conceptual background of a student for later work and of providing mathematical experiences interesting in themselves and more general than those encountered in arithmetic.

CORE OBJECTIVES	NOTES TO TEACHERS
a) to change word phrases to open number phrases and vice versa;	See grade 7 guide on page 42 and review as necessary. This work is very important when considering algebraic problem solving and students require repeated practice in order to become proficient.
b) to evaluate open number phrases by substitution in:	The substitution concept should be reviewed using whole numbers followed by further development in two variable expressions using integers, decimal fractions and fractions. Students need much practice in this work. One of their greatest difficulties is using the correct order of operations.
i. one variable expressions,	Students are expected to determine the value of algebraic expressions such as $3t$, $\frac{16}{2t} + 3$ or $7t \div 3$, where t is, for example, 4 or -8.
ii. two variable expressions.	Substitution involving two variable expressions should be developed using positive integers followed by using combinations of positive and negative integers. Examples could include evaluating $a + b$, ab, $2a - b$, $2ab - b$, $\frac{b}{a} - \frac{3}{4}$ or $3ab - b$ where $a = 4$ and $b = 3$ or $a = 1$, $b = -3$.
	Examples involving exponents are not considered a compulsory part of the grade 8 mathematics course.
2. Algebraic Operations	
The student should be able:	
a) to add and to subtract like terms in one variable;	Addition
	Questions could include simplifying by adding $5a + 2a$, $6x + 5x$ or $3t + t + 5$. It is good practice to have students identify the variable (letter) and the coefficient (number in front of the letter) in each term of the algebraic expression. Then, it is easy to establish that adding like terms is accomplished by adding the coefficients together.
	Example
	Add 4b and 2b
	Solution:
	$4b + 2b = (b + b + b + b) + (b + b)$ $= 6b$
	The second step in this solution is included to give students a semi-concrete representation of the problem where "b" might represent "bugs" or "blocks".

CORE OBJECTIVES	NOTES TO TEACHERS
b) to simplify expressions involving both adding and subtracting.	Subtraction Exercises should include questions of the type 9x - 3x, 5s - 4s or 3n - n. Teachers are encouraged to guarantee that the answers have whole number coefficients to begin with. In order to increase difficulty simply allow the answers to attain integral coefficients as in 4m - 6m, t - 7t, etc. NOTE: Students must realize that the coefficient for "a" is understood to be 1. Often, it is beneficial for students to include the 1, especially when beginning a study of algebra. Examples should include the following types of questions: 1. 5a + 4 + 6a + 7 2. 9a + 3 + a 3. a + 7 + 3a - 2 4. 19a - 5 - 12a - 1 etc. Expressions such as 3a + 2 - (2a - 1) should not be considered until grade 9.
3. Solving Equations The student should be able to solve equations in one variable where the variable appears on only one side of the equation.	The word "algebra" is derived from Arabic. The greatest Arab scholar was Al-Khowarizmi. His greatest book, written over 1200 years ago, was titled "al-jabr wa'l mugabalah" and was a book about equations and how to solve them. The whole title was too hard for Europeans so they kept only the word "al-jabr" from which originates our word "algebra". "Al-jabr" actually means to restore something which has been broken. For example, given the equation n - 2 = 7, "al-jabr" allows one to write the equation as n - 2 + 2 = 7 + 2. A number that can be used to replace a variable in an equation to make it true is called a solution of the equation. To solve an equation means to find all of its solutions. For simple equations it is possible to do this by guessing. However, one goal of algebra is to learn how to systematically solve equations without guessing. NOTE: Teachers are encouraged to use only equations which generate integral solutions. The sequence of difficulty should follow from n + 3 = 14 or t - 7 = 4, to 2a + 1 = 5 or 2g - 3 = 7, and to 2x + 3x = 7 + 3. Students need to be encouraged to show a written check of their solution(s). A balanced scale is a good model for understanding algebraic equations because the reasoning in balancing a scale is identical with the type of reasoning used to solve equations. "Balancing scales" problems are available in many mathematics puzzle books.

CORE OBJECTIVES

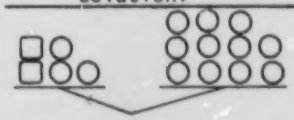
NOTES TO TEACHERS

Consider the following activity as one way to relate the steps in solving an equation to balancing a scale.

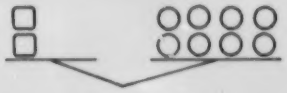
Example:

How many circles would balance one square on the following set of scales?

Solution:



$2x + 3 = 11$



(remove three circles from each side of the balance)

$2x = 8$ (subtract 3 from each side of the equation)

(remove half the contents from each side of the balance)

$x = 4$ (divide each side by 2)

Teachers must stress that all the equations above on the right have the same solution because they are equivalent. In general, any equation can be transformed into an equivalent one by performing any one of the following operations:

1. Adding the same quantity to each side.
2. Subtracting the same quantity from each side.
3. Multiplying each side by the same quantity other than zero.
4. Dividing each side by the same quantity other than zero.

Addition and multiplication are necessary in equation solving. However, subtraction and division need not be used if inverse operations are stressed. For example, the equation $4x = 32$ can be solved by dividing both sides by 4 or multiplying both sides by $\frac{1}{4}$.

These operations can be used to solve equations by changing them into simpler equivalent equations. The idea is to "undo" each operation indicated in the equation by performing the inverse operation on both sides.

CORE OBJECTIVES

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NOTE: Transposition should be discouraged since students tend to see this as an easy method and so wish to use it prior to a thorough understanding of the operations.

Research studies lead to the conclusion that no one approach is right for all students. The successful teacher tries many methods until he or she finds one which seems to work with a particular student or group of students. At this grade level students should be encouraged not to use the so-called "cover-up" technique.

Three methods for solving linear equations are:

1. Inverse Operations Method

   This is the preferred method since it can be used in all cases and it does not employ significant terminology or arithmetic skill.

Example:

Solve $2a - 3 = 5$

Solution:

$2a - 3 = 5$

$2a - 3 + 3 = 5 + 3$ (The additive inverse or opposite of -3 is 3 and both sides of the equation are "balanced" by adding 3.)

$2a = 8$

$\frac{2a}{2} = \frac{8}{2}$ (The opposite of multiplying by 2 is dividing by 2 as shown.)

$a = 4$

Students are expected ultimately to perform mentally the operations in steps 2 and 4 above.

Alternate formats could include:

$2a - 3 \overset{+3}{=} 5 \overset{+3}{}$ or

$$\begin{array}{rcl} 2a - 3 & = & 5 \\ +3 & & +3 \\ \hline 2a & = & 8 \end{array}$$

2. Flow Charts Method

   Cautionary Note: This method may be new for students. Use with care.

CORE OBJECTIVES

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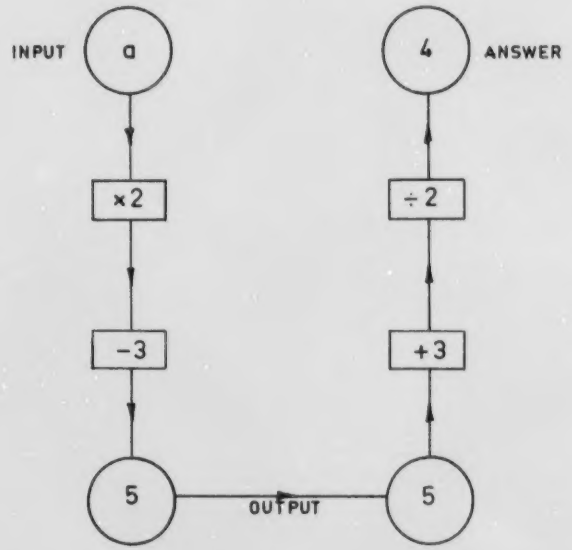
This technique for solving equations is most useful when the equations are relatively simple and the answers are integers. Students become interested because the terminology lends itself to computers and the solution involves a diagram. The mathematics involved uses the order of arithmetic operations and inverse operations.

Example:

Solve the equation 2a - 3 = 5 using a flow chart diagram.



3. Formal Method

This method should not be used with all students.

Example:

Solve 2a - 3 = 5 using the formal method.

CORE OBJECTIVES

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Solution:

Statements	Reasons
2a - 3 = 5	Given
2a - 3 = 8 - 3	Subtraction fact since 8 - 3 = 5, closure or substitution
2a = 8	Cancellation property
2a = 2 x 4	Multiplication fact since 2 x 4 = 8, closure or substitution
a = 4	Cancellation property

* Suggested materials: Game - Guess My Rule (see Madison Project).

4. Problem Solving

The student should be able to translate a word problem into an equation and then to solve the equation.

See note on Problem Solving on page 19.
See grade 7 guide on pages 43-45.

The importance of algebraic problem solving cannot be over-stressed. Students need much more practice in translating from word problems to algebraic equations than in solving the resulting equations. Teachers are reminded that word problems are at the heart of math anxiety. For this reason teachers are expected to provide an organized framework within which students are to solve word problems.

The following steps may be used:

1. identify the variable and what it represents,
2. translate to obtain the equation,
3. solve the equation,
4. verify through checking that the solution satisfies the original problem,
5. state a conclusion.

Every attempt should be made to include practical application problems in this section of the course. Consumer mathematics and geometry (perimeter, area, volume) are excellent sources of such problems.

Example:

Give an algebraic solution to the following problem. Sue paid $32.80 for a tennis racket and three balls. The price of the racket was $29.50. Find the price of a ball.

CORE OBJECTIVES	NOTES TO TEACHERS
	Solution: Let "b" be the price of each ball. 29.50 + 3b = 32.80 3b = 3.30 b = 1.10 Check: Racket 29.50 Balls = 3 x 1.10 = 3.30 Total is 32.80 Each tennis ball costs $1.10. Problems which do not have a definite arithmetic solution should also be presented. Example: A person earns a monthly salary. At Christmas, she is given an extra month's salary as a bonus. How much does she earn in that year? Possible Answer: 13m where m represents her monthly salary.
IV. RATIONAL NUMBERS 1. Identification The student should be able to identify rational numbers.	Students should be aware that any rational number may be written in the form $\frac{a}{b}$ where "a" and "b" are integers and $b \neq 0$. Rational numbers include integers as well as positive and negative fractions and decimal fractions.
2. Conversions The student should be able: a) to change a fraction to a terminating or a repeating decimal fraction by division or by using equivalent fractions;	See grade 7 guide page 65. Students in grade 8 are expected to achieve proficiency in converting fractions to decimal fractions. Students should note certain patterns when converting fractions to repeating decimal fractions. For example, $\frac{4}{9} = 0.\dot{4}$, $\frac{35}{99} = 0.\overline{35}$ and $\frac{469}{999} = 0.\overline{469}$

CORE OBJECTIVES	NOTES TO TEACHERS
b) to change a terminating decimal fraction to a fraction;	can lead students to discover that $\frac{4}{11} = \frac{36}{99} = 0.\overline{36}$. Students need to be aware of the notations commonly used to indicate repeating decimal fractions [$\frac{2}{3} = 0.666\cdots$ or $0.\dot{6}$ or $0.\overline{6}$; $\frac{4}{7} = 0.\dot{5}71428\dot{}$ or $0.\overline{571428}$].
	Example:
	Convert 0.437 to a fraction.
	Solutions:
	Since 0.437 is read as "437 thousandths" simply write the answer as $\frac{437}{1000}$ or use the Property of One for Multiplication as outlined below.
	$0.437 = 0.437 \times 1$
	$= 0.437 \times \frac{1000}{1000}$
	$= \frac{437}{1000}$
c) to change a repeating decimal fraction to a fraction using the "pattern of nines".	The so-called "pattern of nines" is just a reversal of the pattern previously established. In other words, $0.\dot{4} = \frac{4}{9}$, $0.\overline{35} = \frac{35}{99}$ and $0.\overline{469} = \frac{469}{999}$. These ideas can be expanded to include examples such as $5.\overline{4} = 5\frac{4}{9} = \frac{49}{9}$. Students are not expected to deal with decimal fractions such as $0.2\dot{4}$.
3. Operations with Rationals	
The student should be able to perform the four basic operations using rational numbers.	The work involving positive rational numbers is review. However, some time will need to be spent performing arithmetic computations using negative rationals. Students are expected to master such computations involving decimal fractions. Teachers must realize that undue time should not be spent on arithmetic operations. It is better practice to spend a few minutes each day on arithmetic computations rather than to spend weeks or months drilling only on operations.
	NOTE: Teachers may wish to identify the sets of numbers and how they build from counting numbers to rational numbers. Discussion could center on the need for other sets of numbers.
	See grade 9 guide pages 139-141.

CORE OBJECTIVES	NOTES TO TEACHERS
V. RATIO AND PERCENT	
1. Ratio and Proportion	
The student should be able:	See grade 7 guide page 71. Review as necessary.
a) to set up a ratio or a rate between two quantities;	Students should be made aware of the distinction between ratio and rate. A ratio is a method of expressing a relation involving only numbers. The compared quantities have the same unit of measure so the units cancel each other out. A rate is a method of expressing a relation involving numbers and units as in the example 100 km/hr. Any rate may be expressed in the form of a ratio. Many books combine the two ideas in one set of exercises.
b) to use proportions as equivalent ratios in solving problems.	Proportions should be used to solve application problems. Example: A car goes 20 000 kilometres in a given year. To save wear, the five tires are rotated regularly. How many kilometres will each tire have gone in the year if each tire travels the same distance? Solution: Several methods of solution are possible for this problem. The key to problems of this type is to emphasize the assumptions built into the problem and to encourage students to stick to the problem because the people who lose are not the ones who are wrong but the ones who give up. Notice that four of the five tires are being used at any given time so it is reasonable to take 4/5 of 20 000 and get 16 000 kilometres. Or, reason as follows. If the car travels 20 000 kilometres, then the tires together go 80 000 tire-kilometres. If each of the five tires is used equally then each tire will go 80 000 tire-kilometres divided by 5 tires to yield 16 000 kilometres.
2. Percent	
The student should be able:	See grade 7 guide pages 74-78 and review as necessary. Percent is usually introduced after an understanding of rational numbers named as fractions and decimal fractions has been firmly established. New work includes fractional percents and percents larger than 100%.
a) to define percent as a special ratio;	Percent means "out of 100". Hence, to change a fraction to a percent, change the fraction to an equivalent with a denominator of 100.
b) to convert:	

CORE OBJECTIVES	NOTES TO TEACHERS
i. fractions to percents and vice versa,	Students should deal with percents greater than one first and then with percents which are less than one. Students should be prepared to convert fractions such as $\frac{3}{5}$, $\frac{3}{4}$, $\frac{1}{8}$, $1\frac{1}{2}$, $2\frac{5}{8}$ and $\frac{2}{3}$ into percents. To change a percent to a fraction write the percent over a denominator of 100 and reduce the fraction. The following techniques may be used if the percent is fractional: Method 1 $\frac{1}{2}\% = \frac{\frac{1}{2}}{100}$ $= \frac{1}{2} \div 100$ $= \frac{1}{2} \times \frac{1}{100}$ $= \frac{1}{200}$ Method 2 $\frac{1}{2}\% = \frac{\frac{1}{2}}{100}$ $= \frac{\frac{1}{2}}{100} \times \frac{2}{2}$ $= \frac{1}{200}$ Method 3 $\frac{1}{2}\% = \frac{\frac{1}{2}}{100}$ $= \frac{\frac{1}{2}}{100} \times \frac{\frac{1}{100}}{\frac{1}{100}}$ $= \frac{1}{200}$
ii. terminating decimal fractions to percents and vice versa,	It is suggested that all decimal fraction to percent conversions and vice versa be done in two steps. Change the decimal fraction to a fraction, then change the fraction to a percent or change the percent to a fraction, then change the fraction to a decimal. Remind students that this two-step method may take a little longer but much less memorizing is required.
iii. repeating decimal fractions to percents and vice versa.	Do only with $0.\overline{3}$ and $0.\overline{6}$.
* Suggested materials: 100 grid paper, graph paper, hundred cube tray.	
3. Estimating Percents The student should be able:	Students are expected to apply the process of estimation to all percent problems.
a) to round-off percents to the nearest whole percent;	Teachers are reminded that this type of work provides opportunity to show students that a statement such as, "The unemployment level in Canada is 9%", implies that the actual figure falls somewhere between 8.5% and 9.49% (i.e., $8.5 \leq n < 9.5$).

CORE OBJECTIVES

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b) to estimate a percentage of a number.

Developing the ability to make estimates and to make rapid mental calculations is a developmental way of thinking. Isolated lessons on these skills will not suffice; teaching must be permeated with the idea that estimation is the first step in using numbers. Students need to do numerous examples of the following type.

Example:

Estimate 7.8% of $185.00.

Solution:

Think: 7.8% is about 8%; $185.00 is about $200.
Find: 8% of $200.00 ($200.00 x 0.08 = $16.00). (Hopefully, this will be a mental exercise).
Hence, 7.8% of $185.00 is about $16.00.

4. Percent Problems

The student should be able:

When the relations between rational numbers and percents are understood, learners can begin to develop facility with percent problem solving.

See grade 7 guide pages 76-77 and review as necessary.

a) to calculate a specific percentage of a given number;

b) to calculate what percentage one number is of another number;

Example:

If a survey shows that 8 out of every 90 families plan to purchase a major appliance within two months, what percent, rounded to the nearest whole percent, is this?

Solution:

Method 1:

$$\frac{8}{90} = \frac{a}{100}$$

$$\frac{8}{90} \times 100 = \frac{a}{100} \times 100$$

$$\frac{800}{90} = a$$

$$8\frac{80}{90}\% = a$$

The answer rounds off to 9%.

Method 2:

$$\frac{8}{90} = \frac{8}{90} \times 100\%$$

$$= \frac{800}{90}\%$$

$$= 8\frac{80}{90}\%$$

The answer rounds off to 9%.

CORE OBJECTIVES

c) to calculate a number when a percentage of it is known.

NOTES TO TEACHERS

NOTE: This is a new type of percentage problem and is the most difficult type to solve. Students should first estimate their answer and then compare the calculated answer to the estimate.

Example:

20% of what number is 16?

Solution 1:

Proportion Method:

Let "n" = the number

$\frac{20}{100} = \frac{16}{n}$ (It should be noted here that it is often easier for students to simplify the ratio $\frac{20}{100}$ before attempting to find the solution).

$n = 80$

Answer is 80.

Solution 2:

Fraction Method:

This method involves a direct translation of the problem where "of" means "multiplication" and "is" represents "equal to".

$20\% \times n = 16$

$\frac{20}{100} \times n = 16$

$n = 80$

Therefore, 20% of 80 is 16.

Solution 3:

Unitary Method:

20% of n = 16

1% of $n = \frac{16}{20}$

100% of $n = 100 \times \frac{16}{20}$

$n = 80$. Answer is 80.

CORE OBJECTIVES

NOTES TO TEACHERS

Teachers should attempt to use problems which directly relate to student interest and aptitude. The topic of percent was introduced in grade 7 and is to be reviewed in grade 8. This review should not be a repetition of the work of the preceding grade but should be a new view of the topic. Students in grade 8 should apply equation solving techniques to all equations of the type, $\frac{3}{100} = \frac{n}{60}$. The chief function of learning to solve an equation of the type given is not to find 3% of 60, but instead to learn how to solve a fractional equation and to enrich student understanding of percent by formulating the equations.

A common error which students make when they use percents is to assume that successive rates of increase or decrease are additive. For example, students assume that two successive increases of 10% are equivalent to a single increase of 20%, or that an increase of 30% followed by a decrease of 30% is the same as no change at all.

Example:

Insulating the attic of a house cuts fuel consumption by 30%. Insulating the basement of a house cuts fuel consumption by 45%. Insulating the walls of a house cuts fuel consumption by 25%. "Perfect!" said the owner, "I'll do all three and cut my fuel costs by 30% + 45% + 25% = 100%." Is he correct? If not, what percentage will he really save by doing all these insulating jobs in the house?

Solution:

The owner is not correct or most of us would have stopped using energy some time ago.

In order to compute the percent saving, assume the insulating effect takes place in the order listed in the problem and also assume the fuel costs to be $100.00.

Attic insulation saving is 30% x $100 = $30.
Cost of fuel would be reduced to $100 - $30 = $70 after attic insulation.
Basement insulation saving would be 45% x $70 = $31.50.
Cost of fuel would be reduced to $70 - $31.50 = $38.50 after basement insulation.
Wall insulation saving would be 25% x $38.50 = $9.625 or $9.63.
Now, cost of fuel would be reduced to $38.50 - $9.63 = $28.87.
The actual saving is $100.00 - $28.87 = $71.13 which corresponds to 71.13%, or approximately 71% on the fuel costs.

CORE OBJECTIVES	NOTES TO TEACHERS
5. Applications The student should be able to apply the concepts of ratio and percentage relating to:	Of all the special topics in arithmetic none has more widespread use and direct application than the subject of percent. In spite of the importance of percent, many students fail to master it. Students frequently fail to check their results by first estimating their answers. Students should be trained to read and interpret problems carefully and then estimate the answers, prior to solving the problems. In this section use should be made of newspaper advertisements, sale flyers, catalogues, inflation statistics, and current newspaper or magazine articles.
i. provincial sales tax,	See grade 7 guide page 78 and review as necessary.
ii. commission,	Many salespeople are paid on a straight commission basis. This means that they earn a percentage of the amount they sell. Students should realize that there are other methods of payment for salespeople. Wages may include a regular salary, a salary plus commission or a sliding scale of commissions where the rate of commission varies with the price range or type of product sold. It is suggested that community resources be used whenever possible.
iii. simple interest,	Students are expected to calculate simple interest, given the principal, rate and time using the formula $I = Prt$ where I = interest, P = principal, r = rate of interest as a fraction or a decimal fraction and t = time in years or fractions thereof. Students are not expected to manipulate the formula $I = Prt$ to other forms such as $r = \frac{I}{Pt}$, etc.
iv. discount, v. markup and markdown,	Teachers should attempt to use the terms "discount, markup and markdown" in their correct business form. Retailers usually receive a discount from manufacturers and wholesalers when they buy goods for resale. Discounts include trade discounts, chain discounts (two or more successive discounts) or cash discounts for cash payment or payment within a given time period. In order to stay in business, a retailer must mark up the cost price of the goods. This markup must cover operating expenses and profit. A markdown is sometimes called a customer discount. It is usually offered by a retailer to attract customers, to increase the sale of products, or to sell out discontinued lines of merchandise.
vi. scale drawings, vii. recipes, viii. mixtures.	Students should realize that saying "the markup is 42%" is the same as saying "the selling price is 142% of the buying price".
	* Suggested materials: Newspapers, flyers, catalogues, Consumer Education Learning Modules for the Middle Years and Consumer Education Resource Material for Grades 7-12 (available from Department of Education).

CORE OBJECTIVES

NOTES TO TEACHERS

VI. GEOMETRY

1. Measurement

The student should be able:

a) to perform linear and angular measurement;

Teachers are advised to introduce the concepts contained in the geometry section of this guide at varying points throughout the school year. Geometry should not be taught as a separate unit in the grade 8 mathematics program.

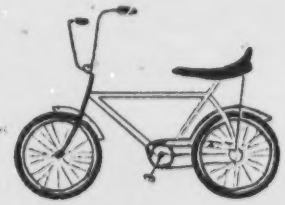
The following geometry development stresses abstracting the properties of geometric figures and relationships among geometric figures from experimentation with many representations of the figures. This is to be done without resorting to precise definitions and deductive methods. Indeed, precise definitions of many seemingly simple geometric ideas are often difficult to arrive at, requiring a subtle foundation of undefined terms, axioms and definitions before arriving at the desired definition. An informal approach, characterized by experiment and intuition, is intended. Geometric concepts have been omitted if they are solely prerequisites for the deductive geometry taught in senior high school.

See grade 7 guide pages 79-88 and review as necessary.

Problems of measurement could well be classified as applied geometry. As with measurement problems in general, the basic idea is the establishment and use of a unit of measure for the particular geometric figure under consideration. For the measurement of length, the units to be learned include millimetre, centimetre, metre and kilometre. In measuring an angle the established unit for grade 8 is the degree. Students are expected to be proficient in measuring and constructing both line segments and angles. The stress is to be placed on using estimating and measuring activities hand-in-hand, throughout the course.

Both oral and written measurement problem exercises should be employed. Students are to become skilled in the use of the appropriate measuring instruments (metric rulers and protractors). Students should be encouraged to take measurements from their environment. Examples could include:

1. measurements associated with a bicycle frame. Time could be spent discussing similarities and differences in various bicycles.



CORE OBJECTIVES

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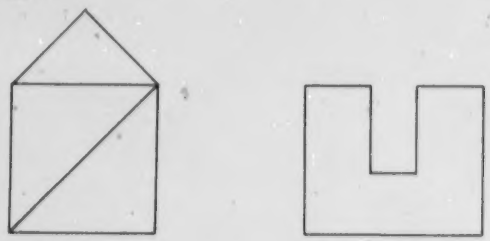
2. determining the angles and linear measures associated with ski runs.

NOTE: Grade 8 students need practice in measuring and constructing angles from 0° to 360°. This is in preparation for the work on circle graphs.

b) to determine the perimeter of any polygon;

See grade 7 guide page 81 and review as necessary.

Students need considerable practice using measurement to help calculate the perimeter for various figures prior to using formulae for such calculations. Students should also have opportunity to determine the perimeter for irregular plane figures such as:



Students must realize that additional lines within a figure do not change its perimeter.

c) to determine the area of any square, rectangle and parallelogram;

See grade 7 guide pages 81-83 and review as necessary.

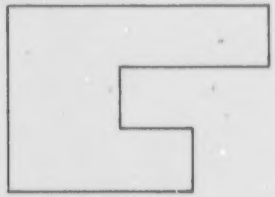
In dealing with the area of a plane figure, the basic unit is the unit square (millimetre square, centimetre square, etc.). In some cases, it may be necessary to begin with a discussion of a square or a rectangle and to approach the concept of area intuitively by the simple process of counting squares. Develop the formula for the area of a rectangle and proceed to the formula for the area of a parallelogram.

CORE OBJECTIVES

NOTES TO TEACHERS

Students in grade 8 are expected to show proficiency in substituting into formulae to determine the area of squares, rectangles and parallelograms. Students need not memorize formulae.

Some time should be spent determining the area of a region which is not rectangular but which can be divided into several rectangular regions. See the example below.



d) to demonstrate an understanding of the concept of volume and to calculate the volume of rectangular solids (i.e., cubes and cuboids);

Teachers are advised that three-dimensional regions present difficulties not encountered in one- or two-dimensions. The volume concept consists of three aspects: internal volume, occupied space and displacement volume. By agreement, a cube is used as the standard unit of volume. (If students disapprove, encourage them to use golf balls as their standard).

Internal volume - the number of cubic units inside the boundary of a spatial region. This is a packing problem employing actual unit cubes.

Problem: Find the measure of the interior region of a shoe box using centimetre cubes.

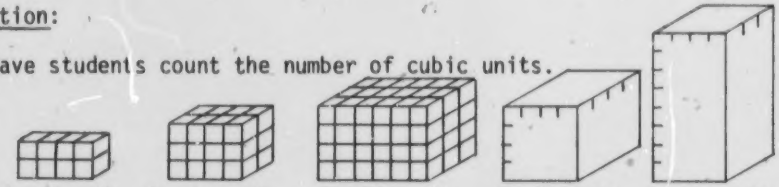
Occupied space - the number of cubic units each figure contains determined by counting.

Example 1:

How many cubic units does each of the following figures contain?

Solution:

Have students count the number of cubic units.

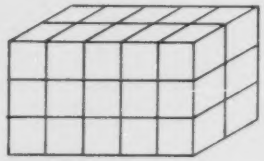


CORE OBJECTIVES

NOTES TO TEACHERS

Example 2:

Determine the volume of the following figure using a formula.



Solution:

Students may discover that volume is equal to the number of layers times the number of rows times the number of cubic units in a row.

Lead students to see that another way to express this formula is to count the number in one layer and multiply this number by the number of layers. Thus, the figure has a volume of $(5 \times 2) \times 3 = 30$ units^3.

Displacement volume - the amount of water displaced by a solid figure represents the volume of the figure. Students have few experiences with displacement volume. This, coupled with the fact that the concept of displacement volume depends on the conservation of the volume of water which is displaced, accounts for students' late acquisition of the displacement volume concept.

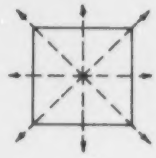
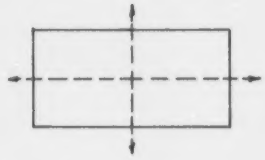
Teachers must not conclude that all concepts involving volume are understood by all grade 7-9 students. A student's understanding of the notion of internal volume occurs prior to that of either occupied or displacement volume.

Traditionally there was a need for differentiating between volume and capacity measures within the Imperial system of measuring as there was little common relationship between, say, cubic inches and gallons. However, there is a common relationship within the SI system since, for example, there are 1000 cubic centimetres in a litre. The common SI units of solid volume measurement are the cubic centimetre (cm^3) and the cubic millimetre (mm^3), whereas the common liquid or capacity units are the litre (L) and the millilitre (mL). The coherence among SI units may eventually eliminate the need for differentiation between volume and capacity measures.

e) to solve problems related to perimeter, area and volume.

See introductory note re Problem Solving on page 19.

CORE OBJECTIVES	NOTES TO TEACHERS
	Students are to become proficient in calculating perimeter and area of rectangular figures and volume of rectangular solids. First, students should solve problems relating to perimeter, then to area, and then to volume. Later exercises should contain problems incorporating perimeter, area and volume together. Additional problems relating to perimeter, area and volume can be found in books containing the following topics: consumer mathematics, geometry, vocational mathematics, or measurement. Practical measurement applications should be sought. For example, the first step in the metric conversion process for Canadian motorists took place in 1977. Road signs for speed limits and distance were converted to kilometres per hour (km/h) and kilometres (km). Since the conversion of road signs in 1977 the term "mileage" is becoming obsolete in Canada. Signs in metres and kilometres accentuate the need for a new word to express odometer readings, distance and gasoline consumption. "Kilometrage" and "metrage" are the terms which are expected to catch on in sectors such as transportation, construction, real estate, surveying, planning, carpeting, forestry, the film industry, consumer and retail trades. On January 1st, 1979, a second step was taken - service stations started selling gasoline and motor oil in litres (L). New car specifications are metric. * Suggested materials: Metric rulers, geopaper, geoboard, centimetre cubes.
2. Quadrilaterals The student should be able:	The term "quadrilateral" simply means "four sides". It is interesting to note that most polygons are named by talking about their angles rather than their sides. We talk about triangles not trilaterals, pentagons not pentalaterals, etc.
a) to identify squares, rectangles, parallelograms and kites;	Review as necessary. Students should be encouraged to identify squares, rectangles and parallelograms in their environment.
b) to determine the lines of symmetry for various quadrilaterals;	See grade 7 guide pages 80, 85, 86.

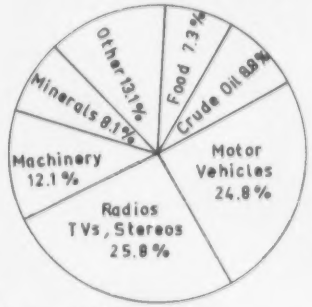
CORE OBJECTIVES	NOTES TO TEACHERS
	A square has 4 lines of symmetry. 
	A rectangle has 2 lines of symmetry 
	How many lines of symmetry does a parallelogram have?
c) to determine the rotational symmetry for various quadrilaterals.	See grade 7 guide page 86 and review as necessary. Students should investigate the symmetries of both regular polygons and three-dimensional figures. Maurits C. Escher (1898-1972) began to produce startling geometric designs about 1935. Escher's prints demonstrate the possibilities of combining mathematics and art. Some of the mathematical ideas included in Escher's work include symmetry, limits, perspective, tessellation and assorted geometric oddities. Since students find Escher's work fascinating, teachers are encouraged to assign student projects relating to optical illusions, tessellations or the art of M.C. Escher at different times in their program.
3. <u>The Circle</u> The student should be able: a) to identify the centre, radius, diameter, arc, sector, and circumference of a circle;	Circles require a new set of terms and definitions. Students need not learn precise definitions. An understanding of the definitions is sufficient. In discussing the circle a teacher should use the terms "centre", "radius", "diameter" and "circumference" frequently. This will help students to become familiar with these terms. A distinction should be made between a circle and

CORE OBJECTIVES	NOTES TO TEACHERS
	a circular disk.
b) to name related three-dimensional objects such as spheres, cylinders and cones;	
c) to draw a circle using compasses given a specific length for the radius or the diameter;	Students need considerable practice in using compasses. In order to give students sufficient practice in the use of compasses, encourage them to create patterns and designs using compasses and ruler. Students should attempt to locate the centre for a given circle. This is a relatively simple problem if the circular disk is made of paper. Cut out the figure and fold to determine two diameters. Where the diameters cross marks the centre. However, this is a more difficult problem if the circle forms a solid circular disk. Folding is not possible. See if students can discover methods for locating the centre of such a circle.
d) to determine the ratio of the circumference of a circle to its diameter and to establish the circumference formula;	Prior to establishing the circumference formula for a circle, students need to explore other methods for finding the distance around circular objects. Encourage students to measure: 1. the distance around a garbage can lid using string and a metre stick. 2. the distance around a tire with only a metre stick. (First, align a mark on the tire with a mark on the ground to indicate a starting point. Roll the tire in a straight line for one complete revolution and make a second mark on the ground. Use the metre stick to measure the distance between the marks on the ground and the student will have found the distance around the tire). Students should be led to discover, through measuring the circumference and the diameter of different sizes of cans or circular disks, that $\frac{C}{d} = \pi$ and hence $C = \pi d$ where C = circumference and d = diameter of the circle. Many such measurements lead to an approximation for π . NOTE: Emphasize that it is customary to use an approximation of π . In grade 8 students may substitute 3.14 or $\frac{22}{7}$ for π but be certain students realize that these values are merely approximations of π . Students should be made aware of the importance of π and that it is a "new" type of number (i.e., irrational).
e) to determine the area of a circle;	The concept of the area of a circle should be developed rather than stated. One method of doing this is by drawing circles on graph paper and then counting squares in order to approximate the area. Using graph papers with different sized squares could lead to a discussion of relative accuracy.

CORE OBJECTIVES	NOTES TO TEACHERS
	Next, develop the relationship for finding the area of a circle. The development is the important aspect. Students need not be required to memorize formulae. However, students are expected to be able to use specific formulae in their mathematics activities. Students should also be taught how to find the formulae they need from a textbook. In order to show that the formula for area contains "π", students could experimentally determine the value of $\frac{A}{r^2}$ for several circles.
f) to determine the surface area of right circular cylinders:	
i. without ends,	This may be determined by opening a tube and then determining the area of the resulting rectangle.
ii. with ends;	This may be determined by opening a can and determining the area of its parts.
g) to interpret circle graphs;	Before students construct their own circle graphs they need practice interpreting the information found in circle graphs. This interpretation is dependent upon estimation, angular measurement, ratio, and percent knowledge. Questions should seek to determine a student's knowledge in each of these concept areas. Correct interpretation of circle graph data is considered more important than the ability to construct accurate circle graphs. The ability to read and to interpret graphs is a necessary basic skill needed for functioning in today's society. The graphical information displayed in magazines, newspapers, books and on T.V. is almost limitless. However, if the examples used in class are not related to the experiences of the students, the examples will become meaningless drill. Social Studies books and Statistics Canada produce excellent circle graphs. Examples could include the following:

ROBIN'S READING LIST

CANADIAN IMPORTS (1976)



(Statistics Canada)

Appropriate questions could be posed for each of these graphs.

CORE OBJECTIVES

h) to construct circle graphs.

NOTES TO TEACHERS

Stress that circle graphs show the relationship of various parts to the whole: the various parts are represented by sectors of the circle and the whole by the area of the circle itself.

Students should be encouraged to follow these steps in constructing a circle graph:

1. find the fractional part for each item;
2. find the number of degrees for each item or section (fraction of 360^0);
3. use a protractor to measure central angles for each sector of the graph;
4. label each sector - name and fraction or percent;
5. give the graph a title.

Example:

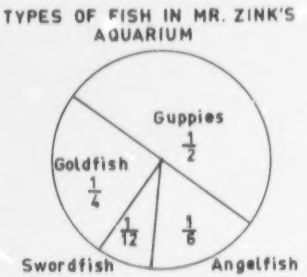
There are 4 angelfish, 2 swordfish, 6 goldfish, and 12 guppies in Mr. Zink's aquarium. Prepare a circle graph showing the types of fish in Mr. Zink's aquarium.

Solution:

Step 1: There are 4 + 2 + 6 + 12 = 24 fish in the aquarium. Angelfish = $\frac{4}{24} = \frac{1}{6}$ of the total; swordfish = $\frac{2}{24} = \frac{1}{12}$ of the total; goldfish = $\frac{1}{4}$; guppies = $\frac{1}{2}$.

Step 2: The sector used to represent angelfish must measure $\frac{1}{6} \times 360 = 60^0$. Other sectors work out to be swordfish 30^0, goldfish 90^0 and guppies 180^0.

Steps 3-5: Prepare a graph as shown below.



CORE OBJECTIVES

NOTES TO TEACHERS

NOTE: Circle graphs usually identify sectors with percents. Occasionally, students could use fractions instead. This would provide an experience in comparing fractions and percents.

4. Motion (Transformational) Geometry

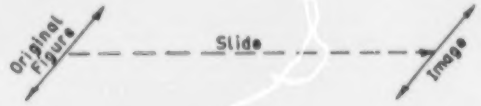
The student should be able:

a) to identify and to illustrate slides (translations), flips (reflections), turns (rotations) and combinations of these rigid motions;

See grade 7 guide pages 89-91 and review as necessary.

b) to identify parallel lines as slide images;

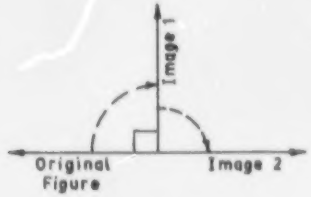
Two lines are parallel if one is the image of the other under a slide.



NOTE: The slide arrow may be located anywhere in the plane but must show correct direction and distance.

c) to identify perpendicular lines as a quarter turn about the point of intersection.

Two lines are perpendicular if one is the image of the other under a quarter turn about the point of intersection.



* Suggested materials: Graph paper, mirrors, Mira, geoboard, geopaper, tracing paper, Game - Transformation Rummy.

If time permits, teachers could extend the study of motion geometry to include a study of the properties of motions.

CORE OBJECTIVES	NOTES TO TEACHERS
	Through experimentation the student should discover the following: 1. Properties of Slides (Translations) a) a line segment and its image are congruent and parallel; b) the line segments joining points of a figure and the images of those points are congruent and parallel; c) a figure and its image are congruent. 2. Properties of Flips (Reflections) a) a figure and its image are congruent; b) each point and its image are equidistant from the reflection line; c) the reflection line is perpendicular to the line segment joining a point and its image; d) a point on the reflecting line coincides with its image (i.e., is "fixed"); e) a figure and its image have reverse orientation. 3. Properties of Turns (Rotations) a) a figure and its image are congruent; b) the centre of rotation (turn-centre) is fixed; c) the distance of a point on a figure from the centre is the same as the distance of the image of that point from the centre.
VII. COORDINATE GRAPHING	See grade 7 guide pages 94-96. Basically, the new work in grade 8 includes extending the rectangular coordinate system to all four quadrants and labelling the horizontal and vertical axes as X and Y respectively.
1. Coordinate Plane The student should be able to demonstrate an understanding of the terminology associated with the coordinate plane.	Students should be familiar with the following terms: 1. coordinate plane; 2. coordinate axes labelled as X and Y; 3. coordinates or ordered pairs; stress the importance of the (X,Y) order; 4. quadrants; 5. origin.

CORE OBJECTIVES

NOTES TO TEACHERS

2. Locating Points

The student should be able to locate and to plot points in all four quadrants and on the axes.

Students should be prepared to do two types of questions:

1. given a point, correctly write the ordered pair needed to name the point;
2. given an ordered pair, draw a labelled rectangular coordinate system with the point correctly marked.

Motivational ideas include picture plotting, enlargement or reduction of line drawings or pictures, the game of Battleships and map reading.

Another application associated with coordinate graphing would be to complete a table associated with $X \cdot Y = 48$. Any integer from -48 to 48 could be used as the value for X. The corresponding value for Y which in some cases is not an integer, would need to be determined and then the points plotted. Emphasize completing the table of values and not the graphing exercise.

Grade 9

GRADE 9 MATHEMATICS GUIDE

The following mathematics guide identifies the objectives for regular grade 9 students. The grade 9 course is broken into two components; namely, a core section and an options section. The core is a continuation of the grade 7 and 8 courses and constitutes approximately 70% of the grade 9 program. The remaining 30% of the time is to be devoted to the study of option topics. Three out of five of these topics are to be selected according to the interests, abilities and future needs of the students.

INDEX TO TOPICS

A. CORE TOPICS

I. REAL NUMBER SYSTEM
1. Number Operations
2. Real Numbers
3. Pythagorean Relation

II. OPEN EXPRESSIONS
1. Meaning of Open Expressions
2. Classification of Open Expressions
3. Exponents
4. Evaluation of Open Expressions
5. Operations

III. EQUATIONS IN ONE VARIABLE

IV. WORD PROBLEMS

V. CONSTRUCTION GEOMETRY
1. Basic Constructions
2. Angles and Parallel Lines

B. OPTION TOPICS

I. CONSUMER RELATED TOPICS
1. Income
2. Net Income
3. Consumer Expenditures
4. Credit

II. STATISTICS
1. Sampling
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1. Congruence
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IV. ALGEBRA
1. Exponents
2. Scientific Notation
3. Operations

V. COORDINATE GRAPHING
1. Review
2. Linear Equations

CORE OBJECTIVES	NOTES TO TEACHERS
A. CORE TOPICS	NOTE: See the section entitled "Unifying Themes" on pages 18-19. The skills identified therein are of such importance that teachers must emphasize their use at every opportunity. All regular grade 9 students are required to learn the material outlined in the core section of this mathematics guide. In order to make the grade 9 mathematics program as meaningful as possible, teachers are advised to consider using the enrichment topics identified in the grade 7 guide on page 34 and in the grade 8 guide on page 97 on a regular basis. Perhaps, part of a period every few days could be devoted to a study of such enrichment topics.
I. REAL NUMBER SYSTEM	
1. Number Operations	
The student should be able:	
a) to demonstrate proficiency in operations with whole numbers, fractions and decimal fractions;	Arithmetic operations must be reviewed and maintained. A variety of drill techniques should be used throughout the year in order to achieve and maintain proficiency. See grade 8 guide pages 98, 115 and review as necessary. It is not intended that this section constitute the majority of the grade 9 program, even for the weak mathematics student. Teachers could integrate measurement geometry, consumer mathematics, or statistics concepts into this section of their program since much of the mathematics of these topics involves operations in arithmetic.
b) to explain and to illustrate the structure of the set of integers;	See grade 8 guide page 102 and review as necessary.
c) to explain and to illustrate concepts of:	
i. absolute value,	Stress that $\|3\|$ and $\|-3\|$ are equivalent and indicate that the distance on a number line from "0" is 3 units in both cases. Note that this is new work.
ii. additive inverse (opposite);	
d) to perform with proficiency the basic operations on integers;	See grade 8 guide pages 102-107. Review as necessary to achieve proficiency. It is most important that students can operate proficiently with integers prior to a detailed study of algebra. Students whose arithmetic skills are good tend to do well in a study of algebra. Perhaps, this is because they need only concentrate on the algebraic concepts rather than worry about the arithmetic operations when determining solutions to exercises.

CORE OBJECTIVES	NOTES TO TEACHERS
e) to explain and to illustrate the meaning of rational numbers;	See grade 8 guide page 114 and review as necessary. Discussion may include the density of rational numbers. Stress that between any two rational numbers, no matter how close they are, one can always find some new rational number by finding their mean.
f) to change fractions to decimal fractions and vice versa:	See grade 8 guide pages 114-115 and review as necessary. Emphasize that rational numbers can be expressed either as terminating or repeating decimal fractions. Discuss an example, such as $\frac{7}{23}$, that has a very long repeating block. Begin the long division process and have the students establish if the process will ever give a repeating decimal. (Look at the remainders at each stage in the division process. The remainder must always be less than the divisor, 23. This means that, at most, 23 different remainders are possible. Thus, at least one remainder must repeat after 23 divisions. It should be fairly clear that the whole process will then repeat. If the remainder becomes 0 the division process will terminate).
i. fractions to terminating or repeating decimal fractions, ii. terminating decimal fractions to fractions, iii. repeating decimal fractions to fractions;	New work should include converting decimal fractions of the form $0.2\overline{7}$ or $0.2\overline{41}$ to fractions. Example: Convert $0.2\overline{41}$ into a rational number in the form $\frac{a}{b}$ where "a" and "b" are integers. Solution 1: Arithmetic Method $0.2\overline{41} = (0.2\overline{41} \times 10) \times \frac{1}{10}$ $= 2.\overline{41} \times \frac{1}{10}$ $= 2\frac{41}{99} \times \frac{1}{10}$ $= \frac{239}{99} \times \frac{1}{10} = \frac{239}{990}$ or $0.2\overline{41} = \frac{2}{10} + \frac{41}{990}$ $= \frac{198}{990} + \frac{41}{990}$ $= \frac{239}{990}$.

CORE OBJECTIVES

NOTES TO TEACHERS

g) to perform the four basic operations with rational numbers;

Solution 2: Algebraic Method

This alternative method is not to be taught to all students. Perhaps it could be used after the core section on solving linear equations.

Let $N = 0.24\overline{1}$ so that $10N = 2.4\overline{1}$ and $1000N = 241.4\overline{1}$

$$\begin{array}{rl} & 1000N = 241.4\overline{1} \\ \text{Subtract} & \underline{10N = 2.4\overline{1}} \\ & 990N = 239 \end{array}$$

$$N = \frac{239}{990}$$

See grade 8 guide page 115 and review as necessary. Emphasis should be on the operations involving negative rational numbers; for example, $\frac{1}{3} + (-\frac{5}{6})$, $\frac{1}{3} - (-\frac{5}{6})$, $0.375 \times (-\frac{1}{4})$ and $-0.2 \div 0.4$.

Time permitting, teachers could extend arithmetic operations to include repeating decimals. One method is to convert each repeating decimal to its fractional equivalent and then to perform the specified operations using the fractions.

Example:

Compute $1.\overline{75} \times 0.\overline{3}$

Solution:

$$1.\overline{75} \times 0.\overline{3} = 1\frac{75}{99} \times \frac{1}{3}$$

$$= \frac{174}{99} \times \frac{1}{3}$$

$$= \frac{58}{99} \text{ or } 0.\overline{58}.$$

NOTE: Teachers are reminded that operations with decimal fractions are considered more relevant than operations with fractions since conversion to metric and the advent of calculators. Undue time should not be spent drilling fraction operations.

h) to solve problems relating to number operations.

See the section entitled "Problem Solving" on page 19. Emphasis should be placed on the assumptions in problem solving situations, on application problems found in the local community and on open-ended problems. Other applications could include percentage problems.

CORE OBJECTIVES

NOTES TO TEACHERS

The following examples are not "type problems" and should be used to initiate discussion. Students at this level are not required to solve such problems.

Example 1:

Bill sold his motor scooter to Tom for $100.00. After driving it around for a few days, Tom discovered it was in such a broken down condition that he sold it back for $80.00. The next day Bill sold it to Herman for $90.00. What was Bill's total profit? Which of the following is the correct solution to this problem?

a) We don't know what the scooter originally cost, so after the first sale, we have no way of knowing whether he made a profit or not. However, since he bought it back for $80.00 and sold it again for $90.00, he clearly made a $10.00 profit.

b) Bill sold his scooter for $100.00 and bought it back for $80.00. He now has the scooter plus $20.00 that he didn't have before so his profit is $20.00. We learn nothing from the next sales because we don't know the scooters real worth, so Bill's profit is $20.00.

c) After Bill buys back the scooter, his profit is $20.00 as just explained. He now sells it for $10.00 more than he just paid for it, making an additional profit of $10.00. Total profit, therefore, is $30.00.

Solution:

One answer is as good as another. In a series of transactions involving the same object, the "total profit" is the difference between what one first paid for it and the amount one has at the end. Because we don't know the original cost of the scooter, we have no way of knowing the total profit.

Example 2:

If an automobile goes from Winnipeg to Regina at a speed of 50 kilometres per hour, how fast should it return to make the average speed for the round trip 100 kilometres per hour?

Solution:

A common error which students make is to confuse an average speed with the average or mean of two numbers. There is a strong temptation to answer by saying, "Return at 150 km/h, since the average of 50 and 150 is 100." However, this is not correct. The correct way to compute average speed is to use the formula on the following page.

CORE OBJECTIVES

NOTES TO TEACHERS

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}}$$

To make the computation easy let us assume that the distance between Winnipeg and Regina is 600 kilometres. The trip to Regina will take $\frac{600}{50}$ = 12 hours. The round trip is a 1200 kilometre trip and if it is to be done at an average speed of 100 kilometres per hour then it must take $\frac{1200}{100}$ = 12 hours. But 12 hours have been spent going to Regina. Therefore, the return trip to Winnipeg must be made in 0 hours, which is obviously impossible.

Thus, there is no return trip speed which is large enough to make the average speed for the round trip 100 kilometres per hour.

Example 3:

What is the net effect of an increase of 20% followed by a decrease of 20%?

Solution:

Assume the item's original value is \$100.
An increase of 20% boosts the value to 100 + (20% x 100) = \$120.
Now, a decrease of 20% drops the value to 120 - (20% x 120) = \$96.
The net effect is a decrease of \$4 which is $\frac{4}{100}$ or a 4% decrease.

2. Real Numbers

The intent of this topic is to show students the development of the number system extending from natural numbers to real numbers. Students should be able to identify a given number as being an example of a natural, a whole, an integer, a rational, an irrational or a real number. Stress that numbers may have more than one name. For example, 5 is natural, whole, integer, rational, and real whereas -3 is integer, rational and real.

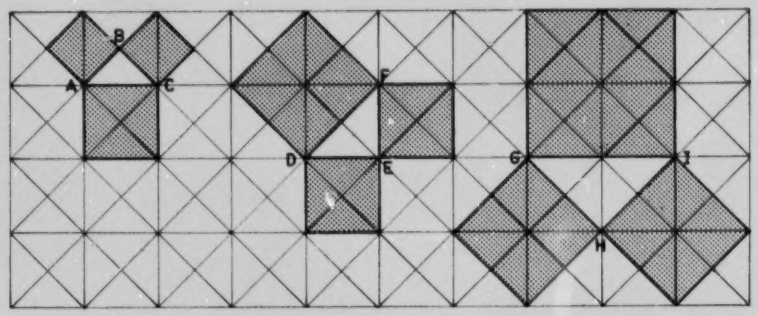
The student should be able:

a) to recognize irrational numbers;

Have students identify decimal fractions that do not terminate or repeat. Examples might include 0.2345678910··· or 0.101101110···. Other irrational numbers include $\sqrt{2}, \sqrt{3}, \sqrt{6}$, π, etc. Stress that the value(s) for $\sqrt{2}$ or π found in a table, or with a calculator, are merely approximations for these numbers.

NOTE: There is no common symbol used in all textbooks to represent the set of irrational numbers. Texts use $\bar{Q}$, Q', S, **J**, etc., to represent the set of irrational numbers.

CORE OBJECTIVES	NOTES TO TEACHERS
b) to examine addition and multiplication of irrational numbers for closure;	Students should realize that the set of irrational numbers is not closed under the operations of addition and multiplication. The following examples could be used. Example 1: $0.1234501234500123450000\cdots$ Add $0.5432165432166543216666\cdots$ $0.6666666666666666666666\cdots = \frac{2}{3}$ This shows that the addition of two irrational numbers can result in a rational number. Thus the set of irrational numbers is not closed under addition. Example 2: $\sqrt{2} \times \sqrt{2} = \sqrt{4} = 2$ This example shows that the product of two irrational numbers can result in a rational number. Thus the set of irrational numbers is not closed under multiplication.
c) to demonstrate an understanding of the definition of the set of real numbers as the union of rational and irrational numbers;	The following sketch shows the development of the real number system. natural → whole → integer → rational, irrational → real
d) to demonstrate an understanding of the relationship between squares and square roots;	Many students have difficulty with these concepts. Students need considerable practice in estimating and computing both squares and square roots. Students should practice identifying approximate answers between specific whole numbers. Example: What is the approximate value of $\sqrt{28}$? Solution: Since $\sqrt{25} = 5$ and $\sqrt{36} = 6$ the $\sqrt{28}$ lies between 5 and 6 and is closer to 5.
e) to find the exact square root of a perfect square;	Students should be able to identify perfect squares up to 144.
f) to find the approximate square root of a natural number using:	Stress that the square root of natural numbers, except those which are perfect squares, are approximate. Also, the value for roots read from tables or

CORE OBJECTIVES	NOTES TO TEACHERS
i. square root tables, ii. a calculator.	calculators is correct, except the last digit is rounded off. It is intended that all students should be able to read answers from a square root table. Every student is not required to use a calculator in order to find square roots. However, many students have a calculator and wish to use it. Two methods of determining the square root with the calculator may be used. Method 1: Use the square root function to determine the square root directly. Method 2: Use a series of approximations by means of successive divisions which can be performed on the calculator. To determine the value of $\sqrt{2}$ start by using 1 as a trial divisor of 2. $1\overline{)2}$ $1<\sqrt{2}$ and $2>\sqrt{2}$, so choose a trial divisor between 1 and 2, say 1.5. $1.5\overline{)2.00}$ (1.3) $1.3<\sqrt{2}<1.5$, so choose the next trial divisor between 1.3 and 1.5, say 1.4 $1.4\overline{)2.000}$ (1.42) $1.4<\sqrt{2}<1.42$ so choose the next trial divisor to be 1.41. $1.41\overline{)2.0000}$ (1.418) The division operation may be repeated as often as necessary in order to obtain a rational number as close to $\sqrt{2}$ as is desired.
3. Pythagorean Relation The student should be able: a) to demonstrate an understanding of the concept of the Pythagorean relation;	Students could be encouraged to discover the so-called Pythagorean relation using patterns. The following pattern was used in China thousands of years ago. About 2500 years ago the Greek mathematician, Pythagoras, saw the pattern. 

CORE OBJECTIVES

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The following steps outline one procedure for discovering the pattern.

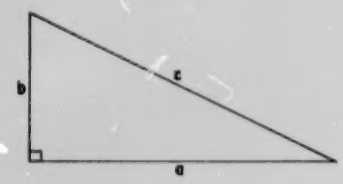
1. What kind of triangles are ΔABC, ΔDEF, ΔGHI? (right triangles or isosceles right triangles).
2. Determine the number of triangular regions in the shaded squares adjacent to each side of the original triangles ABC, DEF, GHI. Complete a table similar to the following.

Area of Squares in Triangular Region Units With Sides Indicated

ΔABC			ΔDEF			ΔGHI		
$\overline{AB}$	$\overline{BC}$	$\overline{AC}$	$\overline{DE}$	$\overline{EF}$	$\overline{DF}$	$\overline{GH}$	$\overline{HI}$	$\overline{GI}$
2	2	4	4	4	8	8	8	16

3. Do you see the pattern? (The square on the longest side contains the same number of triangular regions as the sum of the squares on the other two sides).

Establish the Pythagorean relation: if the measures of the sides of a right triangle are "a", "b" and "c", where "c" is the length of the hypotenuse, then $a^2 + b^2 = c^2$.



Students must be encouraged to include diagrams as part of the solution to any question involving the Pythagorean relation.

Additional suggestions include:

1. develop proofs for the Pythagorean relation;
2. establish that the relation holds only for right triangles;
3. change the letters used to represent the measures of the sides of the right triangle from one question to the next. Otherwise, students may assume "c" always represents the hypotenuse and they may memorize the relation without understanding;

CORE OBJECTIVES	NOTES TO TEACHERS

b) to solve word problems involving the Pythagorean relation;

4. vary the location of the right angle from question to question. Students should deal with right triangles in all possible positions or they may encounter spatial difficulties in the word problems associated with the Pythagorean relation;
5. encourage students to find Pythagorean triples such as 3, 4, 5 or 5, 12, 13.

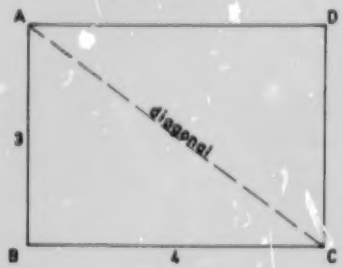
Stress the need for drawing diagrams in problems relating to the Pythagorean relation. Students need practice in identifying two dimensional right triangles from three dimensional real-life situations. Consider the following example.

Example:

A man builds a rectangular granary with sides 3 metres and 4 metres in length respectively. What must the diagonal distance from one corner to another be in order that the corners form right angles?

Solution:

Draw a diagram.



Realize that ΔABC is a right triangle with the lengths of two sides given and the length of the third side, $\overline{AC}$, required.

$$(AC)^2 = 3^2 + 4^2$$
$$= 9 + 16$$
$$= 25$$

Hence, $AC = \sqrt{25} = 5$ metres.

The diagonal distance is 5 metres.

The 3, 4, 5 right triangle relationship is used by construction trades in order to determine whether corners of basements, houses, rooms, etc., are square.

CORE OBJECTIVES

c) to use the Pythagorean relation to locate some irrational numbers on the number line.

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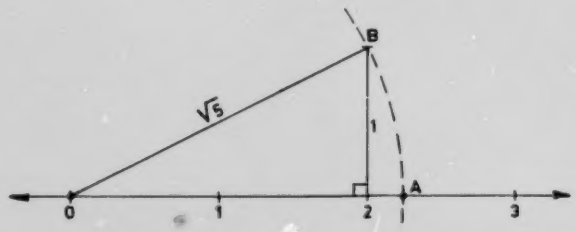
This topic is included in order to show students that irrational numbers can be located on the number line.

Example:

Place $\sqrt{5}$ on the number line.

Solution:

Students must realize that a right triangle with an hypotenuse of $\sqrt{5}$ is required. A triangle with sides of length 1 and 2 will suffice. Proceed as below.



Use compasses to mark A so that $\overline{OA}$ is the same length as $\overline{OB}$. Then, A indicates the position of $\sqrt{5}$ on the real number line.

Summary of the Real Number System

The work on real numbers may be summarized by discussing the limitations of the sets of natural numbers, whole numbers, integers, rational numbers, and irrational numbers. The intent is to examine the real number system as a structure in order to give some idea of flexibility (i.e., which operations are available) as the sets develop. The following table may aid in such a discussion.

CORE OBJECTIVES

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THE PROPERTIES OF NUMBERS

PROPERTY		Set of Numbers					
		Natural	Whole	Integer	Rational	Irrational	Real
Commutative	For addition	X	X	X	X	X	X
	For multiplication	X	X	X	X	X	X
Associative	For addition	X	X	X	X	X	X
	For multiplication	X	X	X	X	X	X
Distributive	For multiplication over addition	X	X	X	X	X	X
	For multiplication over subtraction	X	X	X	X	X	X
Closure	For addition	X	X	X	X		X
	For subtraction			X	X		X
	For multiplication	X	X	X	X		X
	For division				X		X
Identity	For addition (0)		X	X	X		X
	For subtraction (0)		X	X	X		X
	For multiplication (1)	X	X	X	X		X
	For division (1)	X	X	X	X		X
Inverse	For addition			X	X	X	X
	For multiplication (except zero)				X	X	X

II. OPEN EXPRESSIONS

One may ask, "why algebra for all students?" First, the senior high mathematics revision proposes distinct differences between mathematics at the 100 and 101 levels. In order to give students an opportunity to make realistic choices for their immediate future, it is essential that all students are exposed to basic algebraic concepts. Secondly, it is not expected that all students will achieve the same degree of mastery of the core objectives. It should be noted that the aim of the algebraic work in grade 9 is by no means an exhaustive coverage of algebra but rather a sound and progressively improving proficiency of some of the basic concepts and processes. Thirdly, the study of algebra can provide mathematical experiences which are interesting in themselves, more general than those encountered in arithmetic, and pertinent to a review of fundamental operations with integers and rational numbers.

A large share of the difficulties which students encounter in their study of algebra may be attributed to the higher degree of generalization and abstraction than that which they have encountered previously. Also, in contrast to arithmetic, algebra deals more with a conscious examination and study of basic structure than with answers to particular problems. Many mathematics teachers will agree on the importance of proper motivation in the teaching of algebra. With the exception of the very few students who seem to catch on quickly and exhibit an almost natural love of the subject, most others need to have their

CORE OBJECTIVES	NOTES TO TEACHERS
	interest stimulated through suitable teaching techniques and procedures. Any aids which can help make algebraic concepts more explicit deserve attention. Teachers and students could discuss reasons for a study of algebra. Practical real-life applications for algebraic concepts should be sought. Encourage students to assist in these preparations by assigning library research and projects seeking information relating to the practical use of algebra in the community.
1. Meaning of Open Expressions The student should be able to explain and to illustrate the algebraic concepts: i. variable, ii. term, iii. expression, iv. numerical coefficient or coefficient.	Students should not be required to memorize definitions. However, all students need a working knowledge of the terminology associated with any discipline. The ability to identify algebraic terms and numerical coefficients is prerequisite to any work involving algebraic operations. Students must learn to identify "like terms" before adding or subtracting algebraic expressions. If students can readily identify coefficients and then learn to realize that operations involving coefficients are identical to arithmetic operations then algebraic techniques become relatively simple to learn and to remember. Show students that each term of an algebraic expression has a numerical coefficient. Coefficients of 1 usually are omitted. However, at the beginning of the grade 9 course many students benefit from including all coefficients of 1 in their exercises.
2. Classification of Open Expressions The student should be able to identify monomials, binomials and trinomials.	The key to this section is the ability to quickly and accurately identify the number of terms in an algebraic expression. The names for the various expressions are included to give students the opportunity to learn new material over that of previous years and to attempt to standardize formal vocabulary and terminology for the remainder of this mathematics program. The words monomial, binomial, trinomial are easy to remember if you compare them with the number of wheels on a monocycle, bicycle, or tricycle.
3. Exponents The student should be able: a) to demonstrate an understanding of and to illustrate the meaning of the terms "power", "base" and "exponent";	See grade 7 guide page 39 and review as necessary. The student's previous knowledge of these terms must now be extended to algebraic expressions. For example, $5x^3y$ is a monomial or algebraic expression containing one term with the numerical coefficient of 5, the power x^3, whose base is x and whose exponent is 3, and the power y or y^1, with a base of y and an exponent of 1.

CORE OBJECTIVES	NOTES TO TEACHERS
b) to change a numerical power with natural number exponents where the base is either integral or rational to a simple real number;	NOTE: Students must realize that if no exponent is written, it is assumed to be 1. Students often have difficulty understanding and manipulating algebraic powers. In order to help them with this problem, some time should be spent manipulating numerical powers and expressions. Students should become proficient in the following types of questions. Example: Determine the exact numerical value of i) 6^3, and ii) $(\frac{2}{3})^4$. Solution: i) $6^3 = 6 \times 6 \times 6$ or $6 \cdot 6 \cdot 6$ or $(6)(6)(6)$ $= 36 \times 6$ $= 216$ ii) $(\frac{2}{3})^4 = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3}$ $= \frac{16}{81}$ NOTE: If teachers or textbook authors use "x" for a variable then it is suggested that brackets or a dot be used for multiplication.
c) to change a whole number into exponential form;	Students need practice using powers of 2 from 2^1 to 2^{10}, powers of 3 from 3^1 to 3^5, powers of 5 from 5^1 to 5^4 and powers of 10 which employ natural number exponents. Example: Convert into exponential form: i) 125, ii) 36. Solutions: i) $125 = (5)(5)(5) = 5^3$ ii) $36 = (6)(6) = 6^2$ or $36 = (4)(9)$ $= (2)(2)(3)(3)$ $= 2^2 \cdot 3^2$ NOTE: This work simply involves finding and then grouping the prime factors for a given number.

CORE OBJECTIVES	NOTES TO TEACHERS
d) to demonstrate an understanding of the order of operations using exponents.	If students realize that an exponent applies only to the base immediately beside it, there is little difficulty in identifying the value of powers. Students have difficulty in determining the correct answer to $5^2 = 25$, $(-5)^2 = 25$, $-(5)^2 = -25$, and $-5^2 = -25$. This concept is necessary when evaluating open expressions by substitution. Mastery is expected. The sequence of difficulty should proceed from evaluating 5^2, $(-5)^2$, $-(5)^2$, -5^2, $(\frac{3}{4})^4$, $(-\frac{1}{2})^5$, $2^3 \cdot 5^2$, $-2^3 + 5^2$ and on to $5^2 + 2^3$ or $5^2 - 2^3$.
4. Evaluation of Open Expressions The student should be able to evaluate open expressions in one and two variables using substitution.	See grade 8 guide page 108 and review as necessary. Students need much practice in evaluating open expressions. The work in the previous section of this guide is prerequisite here. It is suggested that some standard format be employed for this type of exercise. Stress must be placed upon correct organization and procedure. This section can be used to reinforce arithmetic operations using integers, fractions, and decimal fractions but only after the basic algebraic concepts are understood. Example: Find the value of $3a^2 - 2ab$ if $a = 4$ and $b = 3$. Solution: $3a^2 - 2ab = 3(4)^2 - 2(4)(3)$ $= 3(4)(4) - 2(4)(3)$ $= 48 - 24$ $= 24.$
5. Operations The student should be able: a) to simplify open expressions involving addition and subtraction;	NOTE: It is important that mastery be achieved in this section of the course. Stress the need for identifying "like terms" and then combining the coefficients of the like terms by adding or subtracting. Students need practice in performing algebraic operations involving addition, subtraction, multiplication and division in both horizontal and vertical form. If students wish to use only one format it is suggested that they learn the horizontal one.

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Examples:

Perform the indicated addition.

$(3x + 2m^2) + (5x + m^2)$ or $3x + 2m^2$ Add $5x + m^2$

The level of difficulty for such simplifications should proceed as follows:

1. $5a + 3a$
2. $2a + 4a - 7 + 3$
3. $3a - 5 + 6a - 3$
4. $(5a - 7) + (2a - 8)$
5. $(3a - 5) - (2a - 4)$

b) to multiply a monomial by a monomial;

Rather than have students memorize the laws of exponents, an intuitive approach is preferred. Students have no difficulty accepting that $x^2 = x \cdot x$ and $x^3 = x \cdot x \cdot x$. Therefore, it is reasonable to expect $x^2 \cdot x^3 = (x \cdot x)(x \cdot x \cdot x) = x^5$. This idea can be used to multiply any two monomials.

Example:

$(4a^3)(a^2) = (4 \cdot a \cdot a \cdot a)(a \cdot a)$

$= 4a^5$

After several examples students will see shortcuts. Encourage students to verbalize their shortcuts. Work toward a realization that for monomial multiplication coefficients multiply and exponents of identical bases combine by addition.

NOTE: The multiplication of one monomial by another is critical to all other types of multiplication in algebra. All other forms of algebraic multiplication employ monomial times monomial a number of times.

c) to multiply a monomial by a polynomial;

Example:

$5(2x + 7y) = 5(2x) + 5(7y)$ or $5(2x + 7y) = 10x + 35y$

$= 10x + 35y$

Ultimately, students are expected to perform the distributive step as a mental exercise. Stress should be placed on identifying the number of terms in the polynomial. This tells the number of monomial multiplications which must occur.

CORE OBJECTIVES	NOTES TO TEACHERS
	Some students have difficulty with questions involving a power whose exponent is 1, coefficients of 1, and terms which contain no variable. Examples: 1. $3b\ (2b^4 + 5b^2)$ 2. $3b^2\ (2b^4 + b^3)$ 3. $3b^2\ (2b^4 + 5)$
d) to simplify open expressions involving addition, subtraction and monomial multiplication.	Students are expected to simplify by multiplying and then by combining like terms using addition or subtraction. Students are expected to simplify questions similar to the following. Example: Simplify $4x\ (2x - 7) - 6x\ (3x - 5)$ Solution: $4x\ (2x - 7) - 6x\ (3x - 5) = 8x^2 - 28x - 18x^2 + 30x$ $= -10x^2 + 2x$
III. EQUATIONS IN ONE VARIABLE The student should be able to demonstrate proficiency in solving algebraic equations in one variable.	See grade 8 guide pages 109-113. NOTE: This is an important section of the grade 9 mathematics course. Students are expected to use the additive and multiplicative properties throughout. However, students are not expected to include every step in their solutions. Encourage students to show the work which permits them to make a minimum of arithmetic errors. Also, students should verify answers by substituting in the original equation. Sequence of development: 1. $x + 4 = 7$ 2. $2x = 8$ 3. $2x + 3 = 11$ 4. $2x + 7 = 5x + 1$ 5. $2(x - 3) = 10$ 6. $5(x - 5) - 2(x - 4) = 4$ 7. $4(x - 3) + 5(x - 2) = 3(2x + 4)$ 8. $\frac{x + 3}{4} = \frac{x + 5}{6}$

CORE OBJECTIVES

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In the last example on the previous page, multiply both sides of the equation by the lowest common multiple. Cross-multiplication is not to be encouraged because of the difficulties which students will encounter in future algebra courses, involving questions such as $\frac{2x - 1}{5} = \frac{x}{2} - 7$.

Students should realize that more complex equations reduce to simpler types of equations as illustrated by the following example.

Example:

Solve for x in $\frac{x + 3}{4} = \frac{x + 5}{6}$

Solution:

LCM is 12.

$12(\frac{x + 3}{4}) = 12(\frac{x + 5}{6})$

$3(x + 3) = 2(x + 5)$

$3x + 9 = 2x + 10$

$3x^{-2x} + 9 = 2x^{-2x} + 10$

$x + 9 = 10$

$x + 9^{-9} = 10^{-9}$

$x = 1$

Hence, the answer is $x = 1$.

Check:

L.S. $= \frac{1 + 3}{4} = \frac{4}{4} = 1$

R.S. $= \frac{1 + 5}{6} = \frac{6}{6} = 1$

L.S. = R.S.✓

NOTE: 1. In equation solving, the degree of complexity must be determined by a student's ability.
2. The importance of solving algebraic equations can not be over-emphasized. Mastery of this skill is necessary for continued work in algebra.

* Suggested materials: mathematical balance, Games - Tuf, Algeo (Equation Bingo).

IV. WORD PROBLEMS

One of the best applications of algebra occurs in a study of word problems. In applying algebra, one converts word phrases into numerical or variable expressions and word sentences into equations. "Word problems" reveal how certain numbers relate to one another. After careful reading, a student is required to convert

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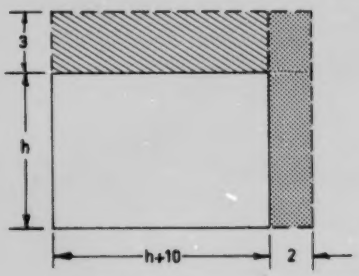
The student should be able:	a written relationship into an equation or algebraic relationship. Once this is done, it is a simple matter to solve the equation and thereby solve the initial word problem. With careful motivation and carefully selected applications this section of the course can be made less difficult for students.
a) to translate word phrases into equivalent open expressions;	Before students can use algebra to solve real-life problems, they will need considerable practice in using variables to translate word phrases and sentences into algebraic expressions and equations. See grade 8 guide page 108 and review as necessary.
b) to solve algebraic word problems.	See note on "Problem Solving" on page 19. It is expected that students will solve numerous word problems by first translating into an algebraic equation and next solving the equation. Emphasis should be placed on estimating and checking answers. There is a tendency for textbook authors and teachers to classify word problems according to type. This often exposes students to only a limited number of different types of questions such as those commonly referred to as "age problems", "coin problems", "rate problems", etc. This practice is not wrong and it is in fact, necessary, particularly with weaker mathematics students. Students need adequate sustained practice. However, every attempt should be made to expand problem solving to include many different problems. Whenever possible non-typical or unusual problems should be used. Diagrams should be drawn wherever possible and students should be encouraged to seek alternate solutions. Example: A rectangular television screen is 10 centimetres wider than it is high. Another model by the same manufacturer has a screen which is 2 centimetres wider and 3 centimetres higher and which has 240 square centimetres more area. Find the dimensions of the first television screen. Solution: Draw a diagram. Let the height of the original television screen be "h" cm as shown. Then, the original width is h + 10 centimetres. The equation is derived from the fact that the area of the two shaded regions equals 240 cm^2. 

CORE OBJECTIVES	NOTES TO TEACHERS

$3(h + 10) + 2(h + 3) = 240$
$3h + 30 + 2h + 6 = 240$
$5h + 36 = 240$
$5h = 204$
$h = \frac{204}{5} = 40.8$

Check:

L.S. $3(40.8 + 10) + 2(40.8 + 3) = 152.4 + 87.6$
$= 240.0$
R.S. $= 240$
L.S. $=$ R.S.✓

The original T.V. screen has dimensions of 40.8 cm by 50.8 cm.

NOTE:
1. In setting up equations, correct translation is important, e.g., "a number has 5 added to it" must be written $x + 5$ (not $5 + x$) in order that students are not confused when the operation is non-commutative.
2. Where possible, problems should have practical applications, e.g., science related problems.
3. Problem solving should be maintained and integrated throughout the grade 9 mathematics program.

V. CONSTRUCTION GEOMETRY

Geometry concepts should be integrated throughout the grade 9 mathematics program. Some teachers may wish to use one class per week for geometry.

Students need compasses and a straightedge for this section of the course.

1. Basic Constructions

The student should be able:

a) to perform the following basic constructions using only compasses and straightedge:

i. to construct the perpendicular bisector of a line segment,
ii. to bisect an angle,
iii. to copy an angle,
iv. to construct a perpendicular to a line through a point on the line,
v. to construct a perpendicular to a line through a point not on the line;

Compasses are used for drawing arcs and measuring line segments while the straightedge is used for drawing line segments. Wherever possible, activities should be kept open-ended.

Reasons for a study of construction geometry include:

1. serving as a creative activity;
2. producing drawings which can be pleasing to the eye;
3. providing a better understanding of geometric figures;
4. developing psychomotor skills;
5. encouraging students to seek the names used to describe patterns or geometric figures.

NOTE: Students could use transformations in their study of construction geometry. Suggested materials for this activity include Mira and tracing paper.

Students should be encouraged to explore applications of the basic constructions. Consider the examples on the following page:

CORE OBJECTIVES

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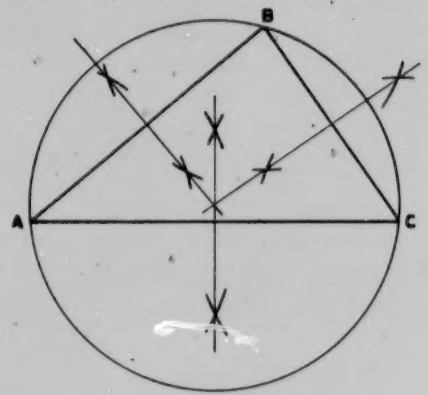
1. the circumcentre of a triangle;

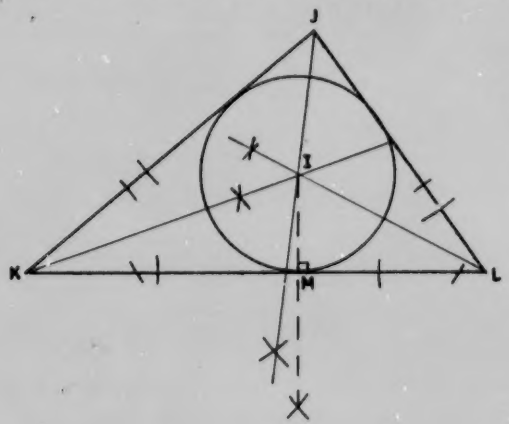
Encourage students to discover that the perpendicular bisectors of the sides of a triangle meet in a single point. Once this is determined see if students can arrange to have the point of intersection fall inside, outside, or on the triangle.



Find the perpendicular bisector of each side of the triangle. The point where the bisectors meet is the circumcentre (i.e., centre for the circle which passes through each vertex of the triangle). The circle is said to be circumscribed about the triangle.

2. the incentre of a triangle;

What happens when you bisect each angle in a triangle?



Bisect each angle in a triangle. The angle bisectors meet at a point called the incentre of the triangle (i.e., centre for the circle which touches the three sides of the triangle).

CORE OBJECTIVES

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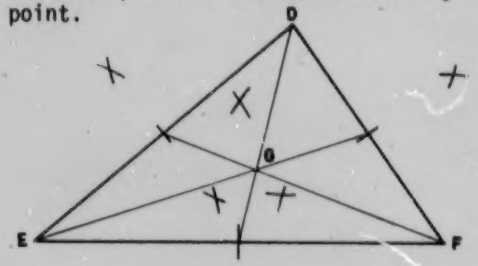
3. the centroid of a triangle;

Find the point in a cardboard triangle where it will balance on a pencil point.



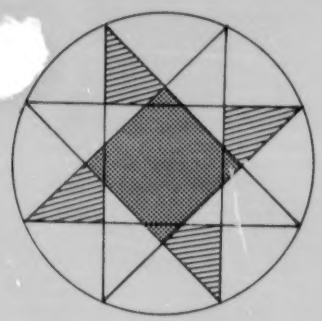
Join the midpoint of each side of a triangle to its opposite vertex. These lines meet at the centroid.

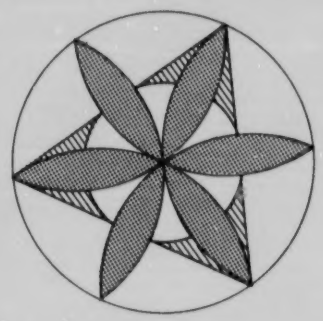
4. the orthocentre of a triangle.

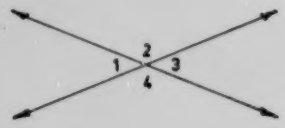
The orthocentre of a triangle is found at the point of intersection of the altitudes of a triangle.

Other construction exercises could include determining the centre for a circle; constructing polygons such as a hexagon (6 sides), an equilateral triangle, a dodecagon (12 sides), a square, an octagon, and a pentagon (5 sides) within a circle; and making tangent circles within an angle.

In addition students should be encouraged to create drawings and challenging designs which involve the basic constructions. Consider the following examples.





CORE OBJECTIVES	NOTES TO TEACHERS
	References: Armour, C. Geometry, Gage Educational Publishing Ltd., 1974. Allen, C.E. Supermath Number 1 (Mathematics in Modules), Rand McNally and Company, 1976.
b) to construct, without the aid of a protractor, angles of: i. 60^0 (hence 30^0, 15^0, 120^0), ii. 90^0 (hence 45^0, $22\frac{1}{2}^0$, 135^0), iii. a composition of the above angles;	In order to obtain a 60^0 angle construct an equilateral triangle. An angle of 150^0 could be constructed as $90^0 + 60^0$ or $180^0 - 30^0$.
c) to copy triangles;	The student should be encouraged to discover what information is sufficient in order to copy a given triangle. It is important that students ultimately realize that the only conditions sufficient to produce an exact replica of a given triangle are outlined in the next objective on the left. Some students may feel that three angles would produce an exact copy of a triangle. If this happens, encourage the class to construct a triangle using three given angles. Many varieties are to be expected. An overhead projector is excellent for showing that three angles cannot produce a copy of a triangle.
d) to construct triangles given the measures of: i. three sides, ii. two sides and the included angle, iii. two angles and any side, iv. the hypotenuse and any side.	Some informal summary should follow the intuitive approach outlined in notes to teachers for (c) above.
2. Angles and Parallel Lines The student should be able: a) to identify and to demonstrate an understanding of the concept of opposite angles;	Some books refer to "opposite angles" as "vertical angles" or "vertically opposite angles".  Opposite angles are $\angle 1$ and $\angle 3$ or $\angle 2$ and $\angle 4$ in the diagram on the left. Students should discover that opposite angles have the same measure.

CORE OBJECTIVES	NOTES TO TEACHERS
b) to demonstrate an understanding of the relationship between complementary angles and between supplementary angles;	
c) to demonstrate an understanding of the concept of parallel lines;	
d) to demonstrate an understanding of the relationship amongst angles created by parallel lines and a transversal:	An understanding of these concepts, rather than formal definitions and proofs, should be emphasized. Students should measure all angles created by a transversal and a pair of parallel lines in order to determine which angles have the same measure and thus are the same size, etc.
i. corresponding angles,	
ii. interior or exterior alternate angles.	Corresponding angles are $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$, $\angle 4$ and $\angle 8$. Interior alternate angles are $\angle 3$ and $\angle 5$ or $\angle 4$ and $\angle 6$. Exterior alternate angles are $\angle 1$ $\angle 7$ or $\angle 2$ and $\angle 8$.

CORE OBJECTIVES	NOTES TO TEACHERS
B. OPTION TOPICS	Students are required to spend 30% of the time devoted to grade 9 mathematics on the following option topics. Three out of five of these topics are to be selected.
I. CONSUMER RELATED TOPICS	Teachers are encouraged to discuss general principles rather than to concentrate on detailed analyses and arithmetic problems. The use of community personnel and resources should be considered. Since it is constantly necessary to update the information on consumer mathematics few examples will be included in this guide. However, teachers using this option are encouraged to make this a worthwhile yet current topic reflecting "This is Your Life". The overall objective of this unit is to provide an environment where students can recognize the value of mathematics and of education in general. Equally important is the creation of an atmosphere which encourages students to believe in their own ability to learn, to make their own choices, and to accept responsibility for creating their own good life.
1. Income The student should be able to illustrate the following sources of income: i. earned: - salary (hourly, weekly, etc.), - professional fees, - self employment, - commission; ii. other: - welfare, - pensions, - family allowance, - interest, - unemployment insurance, - compensation.	NOTE: It is not intended that students will have a thorough knowledge of all sources of income. Students should concentrate on those sources of income which are of most interest to them. Activities could include: 1. "Let's Pretend" - an oral or written exercise with rather sobering revelations. Students are to select a job based on present qualifications or high school graduation. It soon becomes apparent that such qualifications lead only to minimum-pay jobs. Consequently, such persons cannot afford to live in their desired lifestyles. They should continue to live at home, rely on public transportation if possible, and accept a suitable first job in order to gain some experience; 2. Complete a Job Application - a written exercise to be completed initially without assistance using local job application forms. Discussion should later centre on correct procedures and promoting oneself; 3. Job Interviews - an oral activity preferably undertaken by a local employer; 4. Complete a Social Insurance Application; 5. Complete Other Applications - family allowance, unemployment insurance, compensation, welfare, etc.; 6. Discuss Minimum Wage Rates, Implications of a Shorter Work Week, Overtime Rates, etc. Suggested materials include employment circulars from Canada Manpower and resources from the various levels of government.

CORE OBJECTIVES	NOTES TO TEACHERS
2. Net Income The student should be able to demonstrate an awareness of the major deductions which result in net income.	Deductions which should be mentioned include income tax, Canada Pension, superannuation or company pension, union or professional dues, group life insurance, health insurance, salary continuance insurance, employee charity trust or United Way, savings plan or Canada Savings Bonds, parking, etc. Students should discuss opting in or out of those deductions which are optional. Activities could include: 1. Complete Pay Stub Entries. There are two aspects to this activity. First, discover how deductions are computed. Secondly, perform sample calculations. 2. Net Income - determine percentage lost to compulsory deductions and total deductions. 3. Complete an Income Tax Form - tax forms are available from Revenue Canada, Revenue Building, 391 York Avenue, Winnipeg, Manitoba, R3C OP5, or the Post Office.
3. Consumer Expenditures The student should be able to demonstrate an awareness of the cost of living for: i. food, ii. clothing, iii. housing, iv. transportation, v. recreation, vi. insurance: - property, - automobile, - life, - crop, vii. other.	This section of the unit should set a tone of realism and practicality by letting students take an honest look at the dollar cost of the material things which people need and want. Activities could include: 1. T.V. Game Show - classroom activity in which students guess the prices of common items. Later, students could check the actual prices at the local shopping centre. 2. Housing Project - an examination of cooperative housing, of townhouse rental, of owning a single family home or condominium and of apartment rental. Students should actually seek out the benefits, disadvantages and costs of several forms of housing. Perhaps students could be divided into groups, each responsible for reporting to the class on one form of housing. 3. Owning a Car - examine all costs associated with an automobile. Some students require and want guidance on obtaining a driver's license. 4. Local Insurance - invite an insurance agent to make a class presentation on student determined types of insurance. 5. Displays - have groups of students prepare displays on the previously mentioned activities. * Suggested materials: flyers, catalogues, utility bills, tax statements, brochures from the Manitoba Public Insurance Corporation, the Manitoba Crop Insurance Corporation or private insurance companies, etc.

CORE OBJECTIVES	NOTES TO TEACHERS
4. Credit The student should be able to demonstrate the advantages, disadvantages, and the computations involving: i. cash vs. credit buying, ii. borrowing money.	Assume that savings accounts are getting bigger and bigger, the income tax refund is on its way, the job is steady, and everyone really needs (and wants) to make a major purchase. Activities could include: 1. "Moneychangers" - an investigation and tour of local banking institutions relating to personal loans. 2. Complete a Loan Application. 3. Complete a Credit Card Application. 4. Cost of Credit Buying - a mathematical exercise showing the actual cost of paying for goods over an extended period. 5. Examination of Credit Ratings and Credit Responsibilities. Teachers are encouraged to summarize the topic of Consumer Mathematics with a discussion on budgeting. Students need to begin considering personal cash flow. At this age it is vital to become a wise consumer. Community and parent volunteers should be included in an attempt to teach real world consumerism. For a list of Department of Education consumer resources see page 70. NOTE: Wherever calculations are required, calculators could be used to reduce tedium and to increase interest. * Suggested materials: newspapers, periodicals, Statistics Canada data, employment circulars from Canada Manpower, flyers, catalogues, utility bills, tax statements, income tax forms, insurance brochures, various application forms.

CORE OBJECTIVES	NOTES TO TEACHERS
II. STATISTICS	Why statistics? A knowledge of statistical ideas is of immense value for anyone living in today's world. Statistics is a branch of mathematics closely related to real situations, is a very teachable subject and is particularly suited to discovery methods and individualized projects. It can often be related directly to the interests of students and for this reason it frequently appeals to students, particularly those who have had difficulty with mathematics. Also, a study of statistics is a refresher course in basic arithmetic operations and teachers contemplating the use of calculators have an opportunity to use an excellent student motivator. Students are encouraged to gather their own data and to discuss, analyze and justify any conclusions they deduce from that data. The practical and experimental aspects of the work should not be neglected. Indeed, full student benefit will not be gained if a "do-it-yourself" approach is not adopted as a guiding principle.
1. Sampling The student should be able: a) to distinguish between a census study and a sampling of the population;	A census study attempts to get answers to questions by asking every member of a population. Sampling the population means selecting only a small proportion of the whole population, asking them the questions on which information is sought and from the answers given, estimating what the responses for the whole population would be. Doing a proper job of sampling is difficult.
b) to demonstrate an understanding of random samples and stratified samples;	A sample which is drawn so that every member of the population has an equal chance of being selected for the sample is a random sample. On the other hand, a study, like the Gallup Poll, which reports on, say, low-income, medium-income and high-income families is done by selecting a set number of individuals from each subgroup. This method is called stratified sampling.
c) to identify errors which can be made in sampling the population.	Major sampling errors could include sample size, biased or non-random samples, false reports by the subjects, and interviewer bias. Discussion could centre on the way which sampling errors affect the meanings of the statistical results. Current examples should be used. Students should have opportunity to develop a brief questionnaire and to conduct a census and a sample on a current issue affecting their school or local population.
2. Averages	See grade 8 guide page 99 and review as necessary. The word "average" is used often; one hears of the average man, average marks, average speed, on the average, and so on. (The average family is composed of 1 underpaid male, 1 overworked female and 1.4 underfed children.) "Average" is used here in the sense of "typical" or "normal".

CORE OBJECTIVES

The student should be able to collect data and to determine:

i. mean,
ii. median,
iii. mode.

NOTES TO TEACHERS

Stress that there are three common types of average or measures of central tendency in mathematics. Students may only be familiar with the ordinary average (arithmetic mean or mean). Students should be prepared to identify where the mean, median or mode is used in our society. For example, a shoe store owner might be interested in the mode because it identifies the most commonly sold shoe size in the store.

Students need practice in the calculation of a mean, median or mode using their own data or that provided by the teacher. Every attempt should be made to use interesting and easily understood data.

Some classes may wish to employ the following data:

1. Precipitation (mm);

City	J	F	M	A	M	J	J	A	S	O	N	D
Vancouver	140	120	96	58	49	47	26	35	54	117	138	164
Winnipeg	26	21	27	30	50	81	69	70	55	37	29	22
Halifax	140	119	112	112	109	94	94	96	117	120	143	126
Miami	51	48	58	98	163	187	171	177	241	208	71	43
Calcutta	10	30	36	43	140	297	325	328	251	114	20	5

2. Average temperature (^{o}C);

City	J	F	M	A	M	J	J	A	S	O	N	D
Vancouver	3	4	6	9	13	16	18	18	14	10	6	4
Winnipeg	-18	-16	-8	3	11	17	20	19	1[illegible]	6	-5	-13
Halifax	- 3	- 4	0	5	10	14	19	19	16	10	5	- 1
Miami	19	20	21	23	25	27	28	28	27	26	22	20

CORE OBJECTIVES

NOTES TO TEACHERS

3. The Continents

Continent	Highest Point (m)		Longest River (km)	
Asia	Everest	8 848 m	Yangtze	5 980 km
Africa	Kilimanjaro	5 895 m	Nile	6 690 km
N. America	McKinley	6 194 m	Mississippi/Missouri	3 222 km
S. America	Aconcagua	6 960 m	Amazon	6 510 km
Antarctica	Vinson	5 139 m	no rivers - why?	
Europe	El'brus	5 633 m	Volga	3 685 km
Australia	Kosciuko	2 228 m	Murray/Darling	3 750 km

Data sources could include school records, municipal records, Statistics Canada data, physical education data, an atlas, geography textbooks, etc.

In addition, unusual questions could be added.

Example:

B.C. and Thor are playing golf. On the 9th hole B.C. makes a 5 and Thor gets stuck in the sand trap. By the time he gets out and makes it to the hole, Thor has a 15 and is so "mad" he quits. Here are the scores on the 9 holes:

HOLE	1	2	3	4	5	6	7	8	9	TOTAL
B.C.	6	4	7	5	5	3	5	4	5	44
Thor	4	3	6	4	3	3	4	3	15	45

Calculate the three types of averages for each golfer's score. Both B.C. and Thor claim to have better "average" scores. Who, do you feel, is the better golfer if a low score wins in golf?

Solution:

For B.C. the mean = $\frac{44}{9} = 4.\overline{8}$, median = 5, mode = 5.

For Thor the mean = $\frac{45}{9} = 5.0$, median = 4, mode = 3.

Answers will vary for the last question.

CORE OBJECTIVES

NOTES TO TEACHERS

3. Frequency Tables

The student should be able to set up frequency distribution tables.

A very simple frequency distribution exists whenever a person counts or measures and finds more than one item in the same category. For example, here is the result of a survey about the number of classes each week which are devoted to certain subjects:

Subject	Number of Classes/Week or Frequency
Language Arts	6
Mathematics	5
Science	5
Social Studies	4
Others	4

Students are expected to complete frequency distribution tables with columns titled boundaries or intervals, tally and frequency. Students should obtain their own data as well as use teacher-prepared data.

Example:

Joe McFiddick is a heavy smoker. Alice, a friend of Joe's, is a member of GASP and detests smoking. In order to convince Joe that he was smoking too much, she kept track of the number of cigarettes which he smoked during the days of August and September. This data follows:

26, 34, 46, 42, 18, 28, 40, 49, 28, 40, 33, 35, 29, 46, 40, 28, 35, 30, 35, 22, 40, 27, 47, 33, 19, 35, 28, 40, 42, 21, 40, 48, 29, 48, 27, 40, 36, 40, 32, 40, 29, 41, 46, 34, 26, 48, 33, 40, 28, 26, 40, 31, 37, 47, 24, 42, 48, 27, 33, 40, 21.

Complete the following frequency table using the data above. Use a lowest class boundary of 17 and a class width of 4. Include class boundaries or intervals, a tally and the frequency.

Class Boundaries	Tally	Frequency
17 - 21		
22 -		
⋮	⋮	⋮

Incidentally, Joe quit, married Alice and became the national chairperson of GASP.

CORE OBJECTIVES	NOTES TO TEACHERS
	Additional Suggestions re Frequency Distribution Tables 1. Discuss reasons for using frequency tables. 2. Encourage students to tally numbers as they appear in the raw data. 3. Encourage the use of impossible numbers (those which are not elements of the data) for class boundaries. 4. Add additional columns to the table. These may include class mean, class mean x frequency, relative frequency, percent relative frequency, etc. 5. Use the grouped data to determine the approximate mean using the formula, $\bar{x} = \frac{\Sigma(x \cdot f)}{n}$ where $\bar{x}$ = mean, Σ = sum of, x = class median, f = class frequency and n = number of numbers. Students are not required to memorize the formula. 6. Obtain experimental data by rolling a pair of dice and tabulating the frequency of totals from 2 to 12, etc.
4. Interpreting Data The student should be able to interpret the data illustrated in the following graphs: i. bar graph, ii. histogram, iii. line graph, iv. frequency polygon, v. circle graph.	The ability to read and to interpret graphs is a necessary skill in today's society. Current examples should be drawn from newspapers or magazines and realistic, meaningful examples should be included from other subject areas in the grade 9 program. Check for biases. See grade 7 guide pages 91-93. See grade 8 guide pages 129-130.
5. Illustrating Data The student should be able to illustrate data by drawing labelled graphs of the preceding types.	

CORE OBJECTIVES

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III. GEOMETRY

1. Congruence

The student should be able:

a) to identify congruent figures as having the same size and shape;

b) to relate congruence to the study of motion geometry and symmetry;

Congruent figures are exact copies of each other. Discussion could centre on the availability of congruent figures in our environment, how to create congruent figures, and when it is illegal to make congruent copies of things (counterfeit money, copywritten books, forgery, etc.).

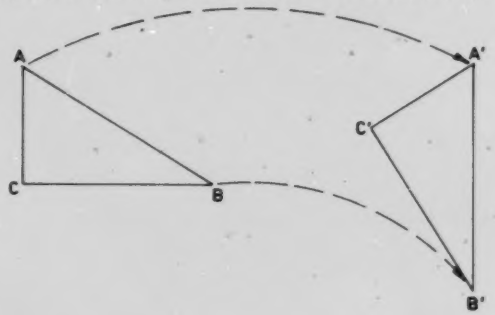
Every attempt should be made to use motion geometry concepts in the study of congruence. Consider sliding, turning and flipping figures in order to obtain congruent copies of original figures. Students should test for congruence by tracing, by folding or by using mirrors or Mira. Attempts should be made to describe examples from everyday life in which geometric figures could be thought of as sliding, turning and flipping. Examples could include:

1. slide or translation - windows in a skyscraper, tiled floors, beehive, post office boxes, chess games, etc.
2. turn or rotation - hands of a clock, circus rides, circular seating arrangements, etc. Mention should be made of point or rotational symmetry.
3. flip or reflection - mirror images, shadows, window or mirror writing, etc. Only when mirror images and shadows are the same size as the original is the relationship a congruence. Mention should be made of bilateral or line symmetry.

Also, teachers are encouraged to have students employ the basic constructions in order to determine the centre of rotation and the line of reflection for two congruent figures. Both constructions involve using perpendicular bisectors.

Example:

Determine, by construction, the centre of rotation for the following figures.



CORE OBJECTIVES

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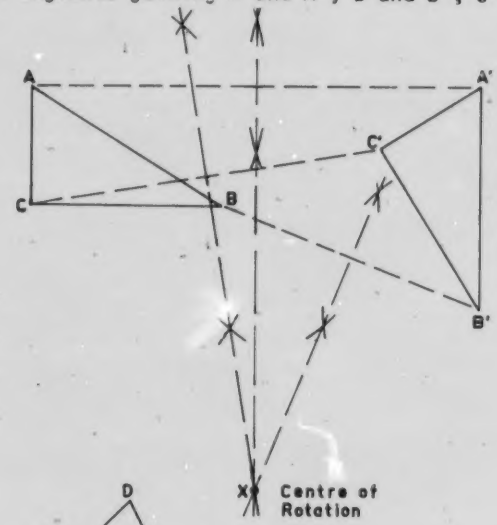
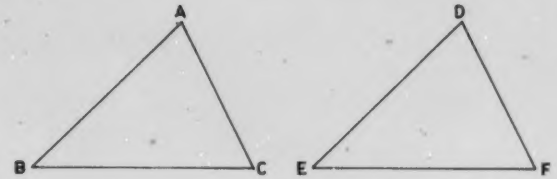
Solution:

The centre of rotation must be at the intersection, X, of the perpendicular bisectors for the segments joining A and A'; B and B'; C and C'.



c) to use congruence notations;



$[A, B, C] \longrightarrow [D, E, F]$ (transformational notation)

$\Delta ABC \cong \Delta DEF$ (conventional notation)

d) to identify corresponding sides and corresponding angles of congruent figures.

It is important that students know that corresponding parts of congruent figures would fit together or correspond if the figures were made to coincide.

2. Polygons

The student should be able:

a) to name polygons according to the number of sides;

Number of Sides	Name of Polygon
3	triangle
4	quadrilateral
5	pentagon

CORE OBJECTIVES	NOTES TO TEACHERS

Number of Sides	Name of Polygon
6	hexagon
7	heptagon
8	octagon

b) to identify the properties of:

i. triangles,
ii. quadrilaterals,
iii. other regular polygons.

It is useful to have students examine polygons in order to discover properties such as congruent sides, congruent angles, congruent diagonals, bilateral (line) symmetry, rotational (point) symmetry, order of symmetry, etc.

Students should identify properties common to all figures of a specific type. For example, some properties of a square which could be identified are:

1. congruent sides and angles;
2. congruent diagonals;
3. diagonals intersecting at right angles;
4. four lines of symmetry;
5. rotational symmetry of order 4.

A special study of the quadrilateral is suggested. Compare the congruence properties of the square, rectangle, parallelogram, rhombus, kite and trapezoid.

3. Similarity

The student should be able:

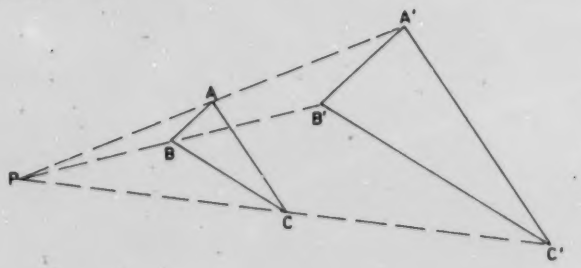
a) to demonstrate an understanding of similarity as an enlargement or as a reduction of figures;

Students must recognize similar figures as those which have the same shape.

b) to use appropriate scale factors to enlarge or to reduce the size of an object;

See grade 7 guide page 95.

The following diagrams illustrate size transformations with centre P and scale factor "K".



$PA^{\prime}$ is 2 times PA (K = 2)

$PA^{\prime}$: PA = 2 : 1 or $\frac{PA^{\prime}}{PA} = \frac{2}{1}$.

CORE OBJECTIVES

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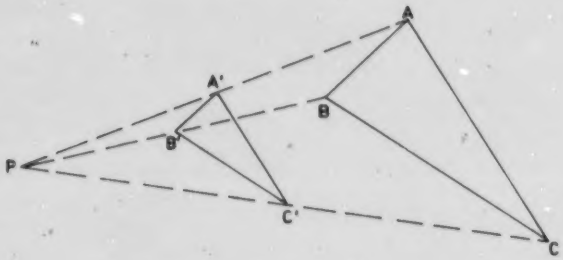


$PA' = \frac{1}{2}$ times PA $(K = \frac{1}{2})$

$PA' : PA = 1 : 2$ or $\frac{PA'}{PA} = \frac{1}{2}$.

Note that for the illustrated examples:

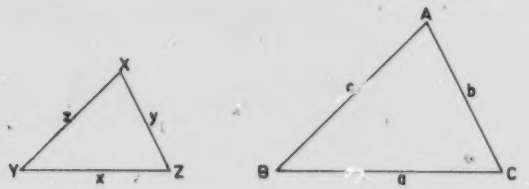
1. the image has the same shape but a different size if $k \neq 1$;
2. the centre P, a point A, and its image A' all lie in a straight line.

c) to discover that similar figures have corresponding angles which are congruent;

d) to discover that similar figures have corresponding sides which are proportional (i.e., in the ratio given by the scale factor).

Similar figures are congruent only if the scale factor is "1".

In grade 9, triangles should be similarly positioned.



If $\Delta XYZ \sim \Delta ABC$, then

$\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$

Students need considerable practice in setting up and then solving proportions.

4. Inductive Geometry

In most of this guide, the program is deductive; that is, a method or algorithm is established and the student then practices the skill until a certain level of competence is achieved. Generally, it is important that the algorithm is committed to memory and is continually reviewed.

This section on intuitive geometry is included so that the student can establish relationships where no methods or rules are given. Thus, the student is to find the rule or relationship for him/her self. It is the establishment not the result

CORE OBJECTIVES

The student should be able to discover the following by experimentation and direct measurement:

i. the number of diagonals in a convex polygon,

ii. the angle sum for a polygon,

iii. the number of intersections of 2 lines, 3 lines, etc.,

iv. in a circle, an angle at the centre is two times an angle at the circumference if they stand on the same arc,

v. an angle on a semi-circle is a right angle,

vi. angles at the circumference subtended by the same arc are congruent,

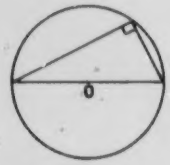
NOTES TO TEACHERS

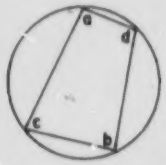
that is important. Students are not expected to memorize the results obtained in such exercises.

This attitude will be new to most students. Many will want to tell others their conclusions. This defeats the prime objective. Teacher input is important and the skilled use of questions and answer techniques is vital. Many students will see the existing relationships but will not be able to verbalize their ideas. The teacher may have to translate in such cases.

It must be stressed to the students that it is the process and not the result that is important.

Most of this content can be done with any ability groups.



CORE OBJECTIVES	NOTES TO TEACHERS
vii. cyclic quadrilateral relationships.	 $a + c = 180^{0}$ or $b + d = 180^{0}$.

* Suggested materials: tracing paper, mirrors, Mira, geoboard, graph paper, photographs, slides, overhead projector, magnifying glass, telescope, microscope, diagram of the eye, zoom lens, camera, pinhole camera, pantograph, protractor, compasses, straightedge.

CORE OBJECTIVES

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IV. ALGEBRA

1. Exponents

The student should be able:

a) to demonstrate an understanding of the concept that $m^0 = 1$ where $m \neq 0$;

Students intending to take 00 Mathematics courses in senior high school should take this topic.

An intuitive approach, which attempts to make the definition meaningful to students is intended. The following procedure could be used.

Write columns of numbers similar to the following on the blackboard.

.	.
.	.
.	.
$2^4 = 16$	$5^4 = 625$
$2^3 = 8$	$5^3 = 125$
$2^2 = 4$	$5^2 = 25$
$2^1 = 2$	$5^1 = 5$
$2^0 = \square$	$5^0 = \triangle$

Point out that as the exponent decreases by one, the corresponding product for the two's is divided by 2. Since $2^1 = 2$ this implies that $2^0 = 2 \div 2 = 1$. The situation is similar for any other natural number and the procedure may be extended to other number systems.

A second approach is characterized by reliance on the field properties and the laws of exponents.

$x^n = x^{n+0}$ (property of zero for addition)

$= x^n \cdot x^0$ (law of exponents)

If $x^n = x^n \cdot x^0$ then $x^0 = 1$. (property of one for multiplication)

Students are not expected to memorize the procedures outlined in this second method.

If students are familiar with the Laws of Exponents a third approach is possible.

Since $\frac{2^4}{2^4} = 1$ and $\frac{2^4}{2^4} = 2^{4-4} = 2^0$ it is correct to say that $2^0 = 1$. This can be

CORE OBJECTIVES	NOTES TO TEACHERS
b) to demonstrate an understanding of negative integral exponents.	generalized for any power. If $x \neq 0$, $n \neq 0$, $\frac{x^n}{x^n} = 1$ and $\frac{x^n}{x^n} = x^{n-n} = x^0$ then $x^0 = 1$. Students are required to know that $m^{-3} = \frac{1}{m^3}$ or $\frac{1}{m^{-3}} = m^3$. Procedures similar to those outlined previously could be used. Some stress should be placed upon the student's ability to evaluate numerical expressions involving negative exponents. Example: Determine the numerical value of 5^{-2}. Solution: $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$. If time permits, students should be introduced to the Laws of Exponents. Appropriate algebraic simplification could follow.
2. Scientific Notation The student should be able:	
a) to rewrite a given number in scientific notation and vice versa;	It is important to stress the need for a notation to express very small and very large numbers. This topic may be introduced using current information relating to electron microscopes and space travel. Some time could be spent discussing applications of one-millionth and one million.
b) to multiply or divide two numbers using scientific notation;	
c) to apply scientific notation and rounding off to estimate the product or quotient of very large or very small numbers.	It is important that students use scientific notation and rounding off to help determine the approximate answer to arithmetic calculations. If students are allowed to use calculators these exercises will be difficult to motivate. However, students should understand the process.

CORE OBJECTIVES

NOTES TO TEACHERS

Example:

Convert each number in the following calculation into scientific notation, round-off all numerical coefficients and then approximate the answer:

$$\frac{649\ 000\ 000\ 000}{0.209\ 600}$$

Solution:

$$\frac{649\ 000\ 000\ 000}{0.209\ 600} \doteq \frac{6.49 \times 10^{11}}{2.096 \times 10^{-1}} \doteq \frac{6 \times 10^{11}}{2 \times 10^{-1}} = 3 \times 10^{12}$$

Emphasize where the solution becomes approximate.

3. Operations

The student should be able:

a) to multiply a binomial by another binomial;

Students must realize that this type of multiplication is not new but rather employs the distributive property and several monomial multiplications.

Many students find it helpful to relate binomial multiplication in algebra to an expanded version of arithmetic multiplication.

$$\begin{array}{r} 43 \\ \times\ 12 \end{array} \quad \text{becomes} \quad \begin{array}{r} 40 + 3 \\ 10 + 2 \\ \hline 80 + 6 \\ 400 + 30 \\ \hline 400 + 110 + 6 \end{array} \text{ or } 516 \qquad \begin{array}{r} a + 3 \\ a + 5 \\ \hline 5a + 15 \\ a^2 + 3a \\ \hline a^2 + 8a + 15 \end{array} \text{ (line up like terms)}$$

Teachers are encouraged to use geometric representations in the initial presentation of binomial multiplication.

Methodolcgies could include:

1. Double Distributive

This preferred method is an application of the distributive property which employs monomial multiplication four times.

Example:

Multiply and simplify $(a + 3)(a + 5)$.

CORE OBJECTIVES

NOTES TO TEACHERS

Solution:

$$(a + 3)(a + 5) = a(a + 5) + 3(a + 5)$$
$$= a^2 + 5a + 3a + 15$$
$$= a^2 + 8a + 15.$$

2. Area Representation

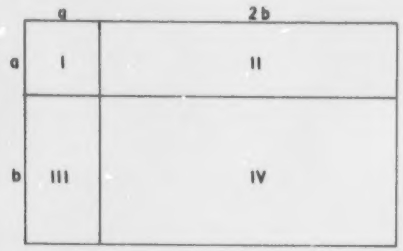
The following illustration should help students to see that, when two binomials are multiplied together there are four terms to consider. Because the area of a rectangle is found by multiplying its length and width, a rectangle whose length and width are known can be used to illustrate binomial multiplication. Each of the four multiplications corresponds to finding the area of a smaller region in the diagram. The answer is the sum of the four areas. Perhaps use an example involving only numbers to begin with.

Example:

Show by an area illustration that $(a + b)(a + 2b) = a^2 + 3ab + 2b^2$.

Solution:

Consider a rectangle whose sides measure "a + b" units by "a + 2b" units as shown below.



Area of rectangle = $(a + b)(a + 2b)$
= area region I + area region II + area region III + area region IV
$= a^2 + 2ab + 1ab + 2b^2$
$= a^2 + 3ab + 2b^2$

CORE OBJECTIVES	NOTES TO TEACHERS

NOTE: The previous activity is suitable for showing a multiplication such as $(a + 3)(a - 1)$. However, care must be taken to ensure that the representation for "a" is larger than "1" and to subtract or remove areas to get the solution.

3. "Let George Do It!"

This method is a takeoff on the so-called "FOIL method" for multiplying two binomials.

Let George Do It!

NOTE: This is included for motivational reasons. Teachers are not encouraged to emphasize the FOIL method. The double distributive method is preferred because it can be adapted for trinomials.

If time permits, students could be introduced to special products such as $(x - y)(x + y)$ and $(a + b)^2$.

b) to divide a monomial by a monomial;

Students are not required to memorize the so-called laws of exponents relating to division. Rather, an intuitive approach is preferred. Students have no difficulty accepting the division or cancellation of like factors in examples such as:

$$\frac{x^5}{x^2} = \frac{x \cdot x \cdot x \cdot \cancel{x} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{x}} = \frac{x^3}{1} = x^3$$

After several examples students see shortcuts. Work toward a realization that in monomial division, coefficients divide as in arithmetic and exponents of identical bases combine by subtraction.

Teachers are reminded to stress that in the division or cancellation of like factors 1's replace all cancelled items.

CORE OBJECTIVES

c) to divide a polynomial by a monomial.

NOTES TO TEACHERS

NOTE: The division of one monomial by another is critical to all other types of algebraic division.

Often, it is helpful to refer to arithmetic operations when introducing algebraic concepts. In arithmetic, fractions could be added in the following way:

$$2 + \frac{3}{4} = \frac{8}{4} + \frac{3}{4} = \frac{8 + 3}{4} = \frac{11}{4}$$

Reverse the process to get: $\frac{11}{4} = \frac{8 + 3}{4} = \frac{8}{4} + \frac{3}{4} = 2 + \frac{3}{4}$. Now, use this idea when working with algebraic division exercises.

Examples:

Perform the appropriate division

1. $\frac{5a + 10b}{5} = \frac{5a}{5} + \frac{10b}{5} = 1a + 2b$ or $a + 2b$.

2. $\frac{6a^3 - 8a^2b + 2a}{2a} = \frac{6a^3}{2a} - \frac{8a^2b}{2a} + \frac{2a}{2a} = 3a^2 - 4ab + 1$.

NOTE: If such division exercises are performed in this manner, students have less difficulty identifying terms which reduce to integers.

If time permits, operations with rational expressions should be introduced.

CORE OBJECTIVES	NOTES TO TEACHERS

V. COORDINATE GRAPHING

1. Review

The student should be able:

a) to demonstrate an understanding of the terminology associated with coordinate graphing including:

i. coordinate axes,
ii. quadrants,
iii. coordinates (ordered pairs),
iv. origin;

b) to plot a point given its coordinates and given a point to write its coordinates.

See grade 8 guide pages 132-133 and review as necessary.

2. Linear Equations

The student should be able:

a) to generate ordered pairs, given the values of "x", using the equation $y = ax + b$ where "a" and "b" may be any real numbers;

This is new work for grade 9 students. Students require practice in determining the numerical value for "y" given a value of "x". The sequence of development should proceed from integral to rational values for a, b and x.

The values of x should be given and a table of values completed as follows:

$y = x + 1$

x	y
2	3
4	5
7	8
.	.
.	.
.	.

or

x	2	4	7	.	.	.
y	3	5	8	.	.	.

The advantage of the vertical table is that it emphasizes the correct x, y order.

CORE OBJECTIVES

b) to graph linear equations, in the form $y = ax + b$, using a table of values;

c) to rearrange any linear equation into the form $y = ax + b$;

NOTES TO TEACHERS

Care must be taken to ensure that students understand:

1. that x and y in an equation represent all points (x, y), on the graph;
2. the benefits of integral values for "x" and "y";
3. why a limited number of points are located on the graph and then a solid line is drawn through these points.

Students learn best if they are exposed to practical applications of the concepts under study. Applications should be drawn from the local community if possible. Many examples of linear relationships and their graphs can be found.

Example:

The linear equation, $y = ax + b$ shows up in the strangest places. Consider the following information gleaned from the RCMP forensic laboratory.

On occasion, the RCMP locate only one or two bones from a human skeleton. In order to facilitate identification it is convenient to know the height of the victim. Consider some of the formulae used for this purpose:

Male

$S = 2.238F + 69.089$ where S = height of a man in cm and F = length of the femur in cm.

$S = 1.225(F + T) + 69.294$ where S = height of man in cm, F = femur, T = tibia.

Female

$S = 2.317F + 61.412$ where S = height of a woman in cm and F = femur.

$S = 1.233(F + T) + 65.213$ where S = height, F = femur, T = tibia.

This example could be used to show students that problems in the real world do not work out nearly as well as most textbook examples.

There are several standard forms for a linear equation. Some authors consider $y = ax + b$ to be standard, others use $Ax + By = C$; while still others employ $Ax + By + C = 0$. Students should be able to convert equations into the form $y = ax + b$.

Example:

Convert the equation $3x + 2y - 7 = 0$ into the form $y = ax + b$ where "a" and "b" are real numbers.

CORE OBJECTIVES

NOTES TO TEACHERS

d) to graph two linear equations, on the same graph, in order to determine the point of intersection.

Solution:

$$3x + 2y - 7 = 0$$
$$3x + 2y = 7$$
$$2y = -3x + 7$$
$$y = \frac{-3}{2}x + \frac{7}{2} \text{ or } y = -1.5x + 3.5.$$

It is suggested that the point of intersection have integral coordinates in order that students can obtain accurate answers. The solution should be checked in both equations.

The following could become items for experimentation and discussion.

1. Identify the uniqueness of solution when two lines cross.
2. The lines of some equations never meet. Look at equations that result in parallel lines.
3. The lines of some equations graph as exactly the same line. Look at equations which graph as the same line.
4. Indicate that there are algebraic methods for finding the point of intersection of two lines. These methods are not part of the grade 9 mathematics course.

Bibliography

VII. BIBLIOGRAPHY

DEPARTMENT OF EDUCATION RESOURCES

A. Books

Education Library (see catalogue),
Main Floor - 1181 Portage Avenue,
Winnipeg, Manitoba.
R3G 0T3
Phone: 786-0221

B. Audio-Visual

1. School Film Services (see catalogue),
   214A - 1181 Portage Avenue,
   Winnipeg, Manitoba.
   R3G 0T3
   Phone: 786-0222

2. School Broadcasts (see catalogue),
   214 - 1181 Portage Avenue,
   Winnipeg, Manitoba.
   R3G 0T3
   Phone: 786-0225

3. Education Library,
   Main Floor - 1181 Portage Avenue,
   Winnipeg, Manitoba.
   R3G 0T3
   Phone: 786-0221

All of the above may be reached via the toll free Wats Line 1-800-262-8848.

TEACHER RESOURCES

The Arithmetic Teacher, The National Council of Teachers of Mathematics, 1906 Association Drive, Reston, Virginia. 22091.

A very useful journal written for elementary mathematics teachers and published eight times per year. Individual or school memberships are available. (K-8)

The Mathematics Teacher, The National Council of Teachers of Mathematics, 1906 Association Drive, Reston, Virginia. 22091.

Written for senior high school and college mathematics teachers. See note above. (7-12)

The Manitoba Mathematics Teacher, Printed by The Manitoba Teachers' Society for the Manitoba Association of Mathematics Teachers.

Published a few times each year. Contains many ideas for classes at all levels. (K-12)

Metric Style Guide, Council of Ministers of Education, Canada, 252 Bloor Street West, Suite S500, Toronto, Ontario. M5S 1V5, 1975.

This publication deals with the correct style to be used in expressing metric terms, units of measure, and their symbols. An explanation as to what each unit represents and how it is to be used in real life situations is included. The guide details the approved standards for use by Canadians as Canada adopts the modernized (SI) metric system. (K-12)

Ashlock, Robert B. Error Patterns in Computation, A Semi-Programmed Approach (2nd Edition), Charles E. Merrill Publishing Co., 1976.

This book is designed for those who want to help students learn to compute. The reader learns to identify error patterns in computation, and he also learns about possible causes of such errors. Specific help for students who need to learn to compute successfully is described. (K-10)

Barnett. Metric Ease, * Creative Publications, 1975.

Suggests a variety of metric measurement activities. (4-8)

Bates, W. W. Mathaction, Copp Clark Publishing Company (Books 1-8), 1969.

A collection of activities and games which provide another alternative to worksheets as a means of drill. Could be used as a motivational aspect. (3-10)

Bell, E.T. Men of Mathematics, Simon and Shuster of Canada Ltd., Markham, Ontario, 1966.

A history.

Biggs, E.E. and MacLean, J.R. Freedom to Learn: An Active Learning Approach to Mathematics, Addison-Wesley Publishing Company, 1969.

Discusses a learning approach involving active learning, discovery methods, and a laboratory approach. (K-8)

Boyle, Pat, Graph Gallery, * Creative Publications, 1971.

Enrichment re ordered pairs, linear equations, and inequalities. (6-10)

Boyle, Pat, Palatable Plotting, * Creative Publications, 1972.

A collection of sets of equations and inequalities which when correctly plotted compose a simple and complete picture. (7-9)

Brandes, L.G. The Math Wizard, Teacher Edition, J. Weston Walch, 1975.

A collection of enrichment suggestions designed to give students the real "flavour" of mathematics rather than a discipline approach. (7-12)

Brousseau, Brother Alfred, Mathematics Contest Problems, * Creative Publications, 1972.

A series of challenging problems. (7-12)

Brown, R.G. et al, Basic Algebra, Houghton Mifflin Company, 1977.

A basic algebra textbook suitable for portions of the content identified in grades 8 and 9. (8-10)

Earle, Richard A., Teaching Reading and Mathematics, International Reading Association, Inc., 1976.

This monograph is written for the mathematics teacher who realizes that success in learning mathematics depends to a large extent on success in reading. It intends to provide the what and the how of teaching reading along with mathematics. (7-12)

Elliott, H.A. et al, Project Mathematics, Books 5 and 6, Holt, Rinehart and Winston of Canada, Ltd.

* This series contains details regarding the games listed in this guide. Additional concrete suggestions abound. (4-9)

Fraser, Don, Power Play! Collier Macmillan Canada Ltd., 1978.

A hockey math book. Questions involving the basic arithmetic operations are handled using current data from the N.H.L. (3-10)

Games, Box 10147, Des Moines, Iowa. 50349.

An excellent magazine containing puzzles, quizzes and games. Produced 6 times per year. (K-12).

Gardner, Martin, Mathematical Carnival, Alfred A. Knopf, New York, 1975.

A bonanza of Martin Gardner's mathematical entertainments from Scientific American. (5-12)

Garvin, Alfred D. Shortcuts, Checks and Approximations in Mathematics, J. Weston Walch, Portland, Maine. 04104, 1973.

Presents alternative ways of doing, checking and estimating basic mathematical operations. (6-10)

Gillespie, N.J. Mira Activities for Junior High School Geometry, * Creative Publications, 1973.

A sequential collection of activities for developing geometric concepts. (7-9)

Herrick, M.C. et al, Mathematics For Achievement/Individualized Course 1, Mathematics for Achievement/Individualized Course 2, Houghton-Mifflin Company, 1972.

Course 1
Book 1 Whole Numbers
Book 2 Sums of Whole Numbers
Book 3 Subtracting Whole Numbers
Book 4 Mastering Multiplication
Book 5 Introducing Number Theory
Book 6 Understanding Division
Book 7 Positive Rational Numbers
Book 8 Applying Number Ideas

Course 2
Book 1 Whole Numbers
Book 2 Number Theory
Book 3 Fractional Numbers
Book 4 Geometry and Measurement
Book 5 Numbers in Science
Book 6 Beginning Algebra
Book 7 Numbers in Business
Book 8 Introducing Statistics

This series could be useful to provide remedial or maintenance activities. (7-9)

Hestwood, Diana and Orf, Earl, Basic Skills In Mathematics, Independent Units. Allyn and Bacon Ltd , 1977.

A series of booklets each dealing with a different topic which could be useful for low-level or remedial mathematics. (7-8-9)

Hestwood, Orf, Huseby, Cross Number Puzzles, * Creative Publications, 1975.

Book 1 Whole Number Operations (4-7)
Book 2 Fractions, Decimals, Percent Operations (5-7)

These books provide practice in basic computational skills. Each puzzle is keyed to a specific objective.

Hogben, Lancelot, The Wonderful World of Mathematics, Doubleday and Company, 1955.

A history. (7-9)

Horst, Paul et al, Sleuthsville, Copp Clark Publishing Company, 1976.

Math mysteries designed to produce super word-problem solvers. (7-9)

Huff, D. and Geis, I. How to Take a Chance, W.W. Norton and Company, Inc., 1959.

Darrell Huff explains clearly and forcefully just what it does mean to take a chance, and shows how to recognize and deal effectively with the probabilities which so importantly affect our judgments and actions. (7-12)

Hull, William P. Attribute Games and Problems, McGraw-Hill Ryerson, 1978.

To develop thinking skills. Activities with classification, logical relationships between attributes and properties of sets are included. (K-12)

Hunter, Figures are Fun, Copp Clark Publishing Company, 1959.

Books 1 to 5 Stories For Problem Solving
A series of five small booklets for grades 4 through 8 designed to enrich the pupils' reasoning ability in mathematics. Interesting settings are built around the basic number processes. Should arouse pupil interest and motivate pupils in daily work. (4-8)

Instructional Aids in Mathematics (34th Yearbook), National Council of Teachers of Mathematics, 1973.

A yearbook containing information about the growing spectrum of available instructional aids, suggestions for selecting and evaluating these materials, and guidance in using them. (K-12)

Jacobs, Harold R. Mathematics A Human Endeavor, W.H. Freeman and Company, 1970.

A textbook for those who think they don't like mathematics. Teachers should also purchase the Teachers Guide which contains both introductory and post lesson activities. (7-12)

Judd, Patterns to Play On the Hundred Chart, * Creative Publications, 1975.

A series of activities to discover patterns in the hundred chart. (4-9)

Knill, George, Mathematics Skillbuilding, SI Metric Edition, McGraw-Hill Ryerson Ltd., 1976.

This booklet is part of the Ryerson Mathematics Program and contains examples and exercises relating to rapid calculation drills and application; equations and formulae; perimeter, area and volume. (7-10)

Marcy et al, Addition and Subtraction With a Happy Ending, * Creative Publications, 1976.

An enrichment book to provide practice in the basic skills of addition and subtraction. (4-8)

Marcy et al, Mathimagination, * Creative Publications, 1973.

Published as six separate booklets (A-F) or as a complete kit in a loose leaf binder. Provides practice in basic skills through a variety of interesting puzzles and activities. (4-9)

McKillop et al, Mathematics Instruction In The Elementary School, Silver Burdett Company, Available: G.L.C. Publishers in Canada, 1978.

A useful teacher reference offering many tested suggestions re techriques and materials to improve teaching - learning. (1-9)

Moore and Williams, Mathematics For Life, Oxford University Press, 1976.

Three series, three books in each series for ages 9 to 15+ of basic or remedial mathematics with relevant applications. (4-9)

Mutart, Alyth et al (Editors), Check and Double Check, Scholars Choice Limited, 1976.

A series of eight arithmetic workbooks for grades 1-8. (1-8)

Network, A Mathematics Series (Hutchinson for Leapfrogs)

1. Action Books - mathematical explorations
   Titles include: Animations, Bands, Chance, Codes, Cubes, Doodles, Folds, Moves, Orders, Pegboards, Prints, Tiles and Dots.
2. Link Books - anthologies of pictures and diagrams
   Titles include: Leads, Leaves, and Links.
3. With Few Words - workbooks which communicate by visual clues.
   Titles include: Blue Book, Green Book, and Orange Book.

Available from: G.L.C. Publishers Limited, 1976.

Network is a series of booklets which provide a wide range of ideas and activities in mathematics. (6-10)

Newton, David, The Use and Misuse of Statistics, J. Weston Walch, Publisher, 1973.

An economical publication containing information pertaining to statistics in everday life, collecting data, statistical analysis of data, and reporting statistical results. (6-12)

O'Daffer and Clemens, Geometry: An Investigative Approach, Addison-Wesley Publishing Company, 1976.

A reference for teachers of geometry at all levels stressing the nature of geometry using the discovery approach and keeping in mind sound pedagogical techniques. Many ideas are new and not a rehash of previous experiences in high school geometry. Topics include polyhedra, tessellations, motion geometry, topology, similarity, number patterns, and measurement using metric units. (4-10)

Pasternak and Silvey, Pattern Block Activities, Book B, * Creative Publications, 1975.

Contains activities for fractions, geometry, area and perimeter. (4-9)

Ranucci, E. R. and Teeters, J. C., Escher-type Drawings, * Creative Publications.

A book containing numerous examples of the works of M. C. Escher and other tessellation drawings. Limited reproduction rights accompany the material. (4-12)

Sawyer, W. W., Mathex, Encyclopedia Brittanica, 1969.

A large collection of discovery or laboratory type activities for all grade levels K-8. (K-8)

Schadler, S., Pic-A-Puzzle, * Creative Publications, 1970.

Illustrate how to create puzzle designs. (1-12)

Sentlowitz, M. and Thelen, J. M., Baseball: A Game of Numbers, Addison-Wesley Publishing Company, 1977.

A book which employs data from American professional baseball. Questions involve statistics in arithmetic operations. (4-10)

Seymour, Dale, Tangramath, * Creative Publications, 1971.

To develop understanding of shapes, similarity, perimeter, and area. (7-10)

Seymour et al, Aftermath Books 1-4, * Creative Publications, 1970.

A series of enrichment activities to reinforce aspects of the curriculum and introduce new ideas to mathematics. Contains cartoons, humour, designs, puzzles, codes, and games. (5-9)

Seymour, Silvey, Snider, Line Designs, * Creative Publications, 1974.

Explains the techniques of curve stitching and designs and contains ideas for creating designs. (6-10)

Seymour and Schadler, Creative Constructions, * Creative Publications, 1974.

Suggests many designs for drawings to be made with straight edge and compass. (5-12)

Shaw, B. R. et al, Mathematics Plus, Houghton Mifflin Company, 1979.

A textbook which emphasizes arithmetic computation using consumer, business and technical applications. (7-9)

Silvani, Harold, Numeration, Creative Teaching Associates, Available: Western School Supplies, Edmonton, Alberta, 1975.

A variety of activities to develop understanding of the number system. (4-9)

Smith, D. E., Number Stories of Long Ago, National Council of Teachers of Mathematics, 8th printing (1977).

Volume 2 in a series "Classics in Mathematics Education". Interesting student readings on the historical development of number systems and patterns. (6-10)

Sobel, Max A. and Maletsky, Evan M., Teaching Mathematics: A Sourcebook of Aids, Activities and Strategies, Prentice-Hall, Inc., 1975.

A book which treats the art of teaching through a series of motivational ideas suitable for many grade levels and abilities, a discussion of various recreational activities appropriate for the classroom, and a collection of laboratory experiments planned for individual or class use. It contains instructions for many inexpensive aids and models useful in teaching, suggestions for developing audio-visual facilities, and extensive listings of resources and references. (7-12)

Stokes and Laycock, Math Activity Worksheet Masters, * Creative Publications, 1974.

A series of graphic aids which are useful in a variety of learning activities. (3-12)

Suydem, Marilyn N. and Reys, Robert E. (Editors), Developing Computational Skills, 1978 Yearbook, National Council of Teachers of Mathematics, 1978.

This yearbook contains articles which express practical consideration for developing computational skills. The authors express divergent viewpoints, gleaned from their own teaching experiences and from working with other teachers. (K-12)

Swienciki, Lawrence W., Adventures with Arithmetic, * Creative Publications, 1974.

Fractions
Decimals
Percent
Algebra

A series of booklets dealing with specific topics. Each contains interesting approaches. (7-8-9)

Wenninger, Magnus J., Polyhedron Models for the Classroom, National Council of Teachers of Mathematics, 1975.

A booklet which describes the history and includes patterns for creating some pleasing geometrical models. (7-12)

Winter, String Sculpture, * Creative Publications, 1974.

Two and three dimensional designs. (5-12)

* Creative Publications are available in Canada from:

Setsco Educational Limited,
567 Clarke Road,
Coquitlam, B. C.
V3J 3X4

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Textbooks

VIII. Textbooks

The following list contains the approved textbooks and the recommended supplementary materials for use in 7-9 mathematics effective September 1979. It is anticipated that changes will occur in this list in the future. For this reason, teachers are reminded to consult the annual publication entitled Approved Textbooks K-12 and The Manitoba Text Book Bureau Catalogue for an updated listing of approved and supplementary materials.

A. Approved Textbooks

Grade 7

Del Grande et al. Math, Book 1. Gage Educational Publishing Limited, 1974.

Elliott et al. Holt Mathematics 1. Holt, Rinehart and Winston of Canada Limited, 1976. (Metric Edition).

Ebos et al. Math Is/1. Thomas Nelson & Sons (Canada) Limited, 1975.

Fleenor et al. School Mathematics 1. Addison-Wesley (Canada) Limited, 1974.

Grade 8

Del Grande et al. Math, Book 2. Gage Educational Publishing Limited, 1974.

Elliott et al. Holt Mathematics 2. Holt, Rinehart and Winston of Canada Limited, 1976. (2nd Printing).

Ebos et al. Math Is/2. Thomas Nelson & Sons (Canada) Limited, 1975. (Metric Edition).

Fleenor et al. School Mathematics 2. Addison-Wesley (Canada) Limited, 1974.

Grade 9

Baxter et al. Mathematics For A Modern World. Book 1. Gage Educational Publishing Limited, 1975. (Revised Metric Edition).

Del Grande et al. Math, Book 3. Gage Educational Publishing Limited, 1974.

Elliott et al. Holt Mathematics 3. Holt, Rinehart and Winston of Canada Limited, 1976.

Ebos et al. Math Is/3. Thomas Nelson & Sons (Canada) Limited, 1976. (Metric Edition).

Nichols et al. Modern Elementary Algebra & Geometry. Holt, Rinehart and Winston of Canada Limited, 1971.

Stein, Edwin I. Fundamentals of Mathematics. Allyn and Bacon Limited, 1976.

Stein, Edwin I. Fundamentals of Mathematics. Macmillan of Canada Limited, 1971. (Canadian Edition).

B. Recommended Supplementary Materials

Brown et al. Basic Algebra. Houghton Mifflin Canada Ltd., 1977 Edition, Grade 9, Teacher's Annotated Edition.

Crawford et al. Achievement in Mathematics. Allyn and Bacon Canada Limited.
Book 1 - Grade 7
Book 2 - Grade 8

Del Grande et al. Math. Gage Educational Publishing Limited.
Book 1 - Metric - Teacher's Commentary, Student's Answer Key
Book 2 - Metric - Teacher's Commentary, Student's Answer Key

Denholm et al. Mathematics for Individual Achievement. Houghton Mifflin Canada Ltd.
Book 7 - Student's Book, Teacher's Annotated Edition
Book 8 - Student's Book, Teacher's Annotated Edition

Dottori et al. Mathematics for Today and Tomorrow. McGraw-Hill Ryerson Limited. Metric. Grade 9.

Duncan et al. Mathematics. Houghton Mifflin Canada Ltd.
Mathematics 7 - Student's Text, Teacher's Edition
Mathematics 8 - Student's Text, Teacher's Edition

Ebos et al. Math Is. Thomas Nelson & Sons (Canada) Ltd. Metric.
Book 1 - Teacher's Edition, Answer Key
Book 2 - Teacher's Edition, Answer Key
Book 3 - Teacher's Edition

Eicholz et al. School Mathematics. Addison-Wesley (Canada) Limited.
School Mathematics 1 and 2 - Teacher's Edition, Workbook, Duplication Masters, Achievement Tests.

Elliott et al. Holt Mathematics. Holt, Rinehart and Winston of Canada Limited.
Books 1, 2 and 3 - Teacher's Editions.

Fraser, D. Powerplay! A Hockey Math Book. Collier Macmillan Canada Limited.

Kahn et al. Pacemaker Practical Arithmetic Series. Copp Clark Publishing Company. Using Dollars and Sense, Working Makes Sense, Money Makes Sense.

Odegard, S.E. World of Metric - Activity Book, Ginn and Company, Grades 7-8.

Park et al. Rapid Calculation and Estimating Skills. Copp Clark Publishing Company.

Sentiowitz et al. Baseball: A Game of Numbers.

Shaw et al. General Math. Houghton Mifflin Canada Ltd.

Vernoort et al. Calculator Activities for the Classroom. Copp Clark Publishing Company. Books 1, 2, 3.